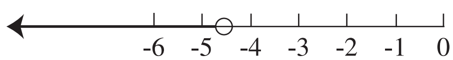
Chapter 2 • Polynomial Equations and Inequalities

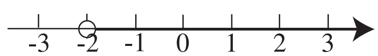
Review of Prerequisite Skills

1. b.
$$\begin{aligned} 3(x-2)+7 &= 3(x-7) \\ 3x-6+7 &= 3x-21 \\ 3x+1 &= 3x-21 \\ 0x &= -22 \end{aligned}$$
There is no solution.

2. c.
$$\begin{aligned} 4x-5 &\le 2(x-7) \\ 4x-5 &\le 2x-14 \\ 2x &\le -9 \\ x &\le -\frac{9}{2} \end{aligned}$$
or $x \le -4.5$



d.
$$\begin{aligned} 4x+7 &< 9x+17 \\ -5x &< 10 \\ x &> -2 \end{aligned}$$



3. $f(x) = 2x^2 - 3x + 1$

b.
$$\begin{aligned} f(-2) &= 2(-2)^2 - 3(-2) + 1 \\ &= 15 \end{aligned}$$

d.
$$\begin{aligned} f\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1 \\ &= 0 \end{aligned}$$

4. $f(x) = x^3 - 2x^2 + 4x + 5$

c.
$$\begin{aligned} f(-3) &= (-3)^3 - 2(-3)^2 + 4(-3) + 5 \\ &= -52 \end{aligned}$$

d.
$$\begin{aligned} f\left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) + 5 \\ &= \frac{53}{8} \end{aligned}$$

5. d.
$$\begin{aligned} &3x^3 - 75x \\ &= 3x(x^2 - 25) \\ &= 3x(x-5)(x+5) \end{aligned}$$

f.
$$\begin{aligned} &x^3 + x^2 - 56x \\ &= x(x^2 + x - 56) \\ &= x(x+8)(x-7) \end{aligned}$$

h.
$$\begin{aligned} &3x^3 - 12x \\ &= 3x(x^2 - 4) \\ &= 3x(x-2)(x+2) \end{aligned}$$

6. e.
$$\begin{aligned} x^2 - 2x - 15 &= 0 \\ (x-5)(x+3) &= 0 \\ x-5 = 0 \quad &\text{or} \quad x+3 = 0 \\ x = 5 \quad &\text{or} \quad x = -3 \end{aligned}$$

f.
$$\begin{aligned} 7x^2 + 3x - 4 &= 0 \\ (7x-4)(x+1) &= 0 \\ 7x-4 = 0 \quad &\text{or} \quad x+1 = 0 \\ x = \frac{4}{7} \quad &\text{or} \quad x = -1 \end{aligned}$$

h.
$$\begin{aligned} x^3 - 9x &= 0 \\ x(x^2 - 9) &= 0 \\ x(x-3)(x+3) &= 0 \\ x = 0 \quad \text{or} \quad x-3 = 0 \quad &\text{or} \quad x+3 = 0 \\ x = 0 \quad \text{or} \quad x = 3 \quad &\text{or} \quad x = -3 \end{aligned}$$

7. b. $3y^2 - 5y - 4 = 0$
$$\begin{aligned} y &= \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-4)}}{2(3)} \\ &= \frac{5 \pm \sqrt{73}}{6} \\ &\doteq 2.3 \quad \text{or} \quad -0.6 \end{aligned}$$

c. $3x^2 + x + 3 = 0$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{41}}{2}$$

$$\doteq 5.7 \quad \text{or} \quad -0.7$$

e. $2x^2 - 5x - 3 = 0$

Solution 1

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{49}}{4}$$

$$= 3 \quad \text{or} \quad -0.5$$

Solution 2

$$(2x+1)(x-3) = 0$$

$$2x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$2x = -1 \quad \text{or} \quad x = 3$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = 3$$

g. $2p^2 - 3p + 5 = 0$

$$p = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{-31}}{4}$$

$$= \frac{3 \pm i\sqrt{31}}{4}$$

i. $2x(x-5) = (x+2)(x-3)$

$$2x^2 - 10x = x^2 - x - 6$$

$$x^2 - 9x + 6 = 0$$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{9 \pm \sqrt{57}}{2}$$

$$\doteq 8.3 \quad \text{or} \quad 0.7$$

Exercise 2.1

2. b. The other factors can be found by dividing $x - 5$ into $f(x)$ then checking factors for the quotient, either by inspection or using the Factor Theorem.

3. If $f(x) = x^3 + 2x^2 - 5x - 6$
and $f(-1) = f(2) = f(-3) = 0$.

then the factors are $(x+1), (x-2)$, and $(x+3)$. This is true since $f(a)$ is the remainder, and in this case, all remainders are zero, giving division that is complete. Also, this is the Factor Theorem.

4. a. $x - 1$ is a factor of $f(x) = x^2 - 7x + 6$ only if $f(1) = 0$.

Since $f(1) = 1^2 - 7(1) + 6 = 0$, then $x - 1$ is a factor.

d. $f(x) = x^3 + 6x^2 - 2x + 3$

$$f(3) = 3^3 + 6(3^2) - 2(3) + 3$$

$$\neq 0$$

Therefore, $(x-3)$ is not a factor of $f(x)$.

f. $f(x) = 4x^3 - 6x^2 + 8x - 3$

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) - 3$$

$$= \frac{1}{2} - \frac{3}{2} + 4 - 3$$

$$= 0$$

Therefore, $(2x-1)$ is a factor of (x).

5. $f(x) = x^3 - 2x^2 - 2x - 3$

a. $f(3) = 3^3 - 2(3)^2 - 2(3) - 3$
$= 27 - 18 - 6 - 3$
$= 0$

b. $x - 3$ is a linear factor of $f(x)$.

c.
$$\begin{array}{r}
x^2 + x + 1 \\
x - 3 \overline{)\, x^3 - 2x^2 - 2x - 3} \\
\underline{x^3 - 3x^2} \qquad\qquad \\
x^2 - 2x \qquad \\
\underline{x^2 - 3x} \qquad \\
x - 3 \\
\underline{x - 3} \\
0
\end{array}$$

The quadratic factor is $x^2 + x + 1$.

6. $g(x) = x^3 - 2x^2 - 5x + 6$

a. $g(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$
$= 0$

b. $x + 2$ is the linear factor of $f(x)$.

c. $x^3 - 2x^2 - 5x + 6 = (x + 2)(x^2 + kx + 1)$
$= x^3 + (k + 2)x^2 + (k + 1)x + 2$

By comparing coefficients, $k + 2 = -2$
$k = -4$.

$\therefore$ the quadratic factor is $x^2 - 4x + 1$.

7. a. Let $f(x) = x^3 - 4x + 3$.
$f(1) = 1^3 - 4(1) + 3$
$= 0$

$\therefore$ $(x - 1)$ is a factor of $f(x)$.
$x^3 - 4x + 3 = (x - 1)(x^2 + kx - 3)$
$= x^3 + (k - 1)x^2 + (-k - 3)x + 3$

Comparing coefficients, $k - 1 = 0$
$k = 1$.
$\therefore x^3 - 4x + 3 = (x - 1)(x^2 + x - 3)$.

b. Let $f(x) = x^3 + 2x^2 - x - 2$.
$f(1) = (1)^3 + 2(1)^2 - (1) - 2$
$= 0$
$\therefore$ $(x - 1)$ is a factor of $f(x)$.
So, $x^3 + 2x^2 - x - 2 = (x - 1)(x^2 + kx + 1)$
$= x^3 + (k - 1)\,x^2 + \ldots$
Comparing coefficients, $k = 3$
$\therefore x^3 + x^2 + x + 1 = (x - 1)(x^2 + 3x + 2)$
$= (x - 1)(x + 2)(x + 1)$.

e. Let $f(y) = y^3 - y^2 - y - 2$
$f(2) = (2)^3 - (2)^2 - (2) - 2$
$= 0$.
$\therefore$ $(y - 2)$ is a factor of $f(y)$.
By dividing, $y^3 - y^2 - y - 2 = (y - 2)(y^2 + y + 1)$.

g. $f(x) = x^4 - 8x^3 + 3x^2 + 40x - 12$
Because the function is quartic and the constant is –12, which presents many possibilities, we use the graphing calculator in $\boxed{\text{VALUE}}$ mode in the $\boxed{\text{CALC}}$ function to establish $f(-2) = f(3) = 0$. Therefore, both $(x + 2)$ and $(x - 3)$ are factors of $f(x)$.
Using the method of comparing coefficients to factor,
$x^4 - 8x^3 + 3x^2 + 40x - 12$
$= (x + 2)(x - 3)(x^2 + kx + 2)$
$= (x^2 - x - 6)(x^2 + kx + 2)$
$= x^4 + (k - 1)x^3 + \ldots$
Since $k - 1 = -8$
$k = -7$.
$\therefore x^4 - 8x^3 + 3x^2 + 40x - 12$
$= (x + 2)(x - 3)(x^2 - 7x + 2)$.

h. Let $f(x)=x^4-6x^3-15x^2-6x-16.$

By graphing, it appears x-intercepts are –2 and 8.

Checking,

$f(-2)=(-2)^4-6(-2)^3-15(-2)^2-6(-2)-16=0$

and $f(8)=(8)^4-6(8)^3-15(8)^2-6(8)-16=0.$

Therefore, both and $(x-8)$ are factors of $f(x)$.

$\therefore x^4-6x^3-15x^2-6x-16$

$=(x+2)(x-8)(x^2+kx+1)$

$=(x^2-6x-16)(x^2+kx+1)$

$=x^4+(k-6)x^3+\ldots$

Comparing coefficients, $k-6=-6$

$\therefore k=0.$

So, $x^4-6x^3-15x^2-6x-16$

$=(x+2)(x-8)(x^2+1).$

9. If x^3+4x^2+kx-5 is divisible by $(x+2)$, then $f(-2)=0,$

or $(-2)^3+4(-2)^2+k(-2)-5=0$

$-8+16-2k-5=0$

$-2k=-3$

$k=1.5.$

10. c. $125u^3-64r^3=(5u)^3-(4r)^3$

$=(5u-4r)(25u^2+20ur+16r^2)$

d. $2000w^3+2y^3=2(1000w^3+y^3)$

$=2(10w+y)(100w-10wy+y^2)$

e.

$(x+y)^3-u^3z^3=(x+y)^3-(uz)^3$

$=(x+y-uz)\left[(x+y)^2+(x+y)uz+u^2z^2\right]$

$=(x+y-uz)\left[x^2+2xy+y^2+xuz+yuz+u^2z^2\right]$

f.

$5u^3-40(x+y)^3$

$=5\left[u^3-8(x+y)^3\right]$

$=5\left[u^3-(2(x+y))^3\right]$

$=5(u-2[2x+y])\left(u^2+2u(2x+y)+4(2x+y)^2\right)$

$=5(u-4x-2y)(u^2+4ux+2uy+4(4x^2+4xy+y^2))$

$=5(u-4x-2y)(u^2+4ux+2uy+16x^2+16xy+4y^2)$

11. Let $f(x)=x^3-6x^2+3x+10.$

Since $x^2-x-2=(x-2)(x+1).$

If $f(x)$ is divisible by x^2-x-2, it must be divisible by both $(x-2)$ and $(x+1)$, that is, $f(2)=f(-1)=0$.

Substituting for x, $f(2)=2^3-6(2)^2+3(2)+10$

$=8-24+6+10=0$

and $f(-1)=(-1)^3-6(-1)^2+3(-1)+10$

$=-1-6-3+10=0.$

Therefore, $x^3-6x^2+3x+10$ is divisible by x^2-x-2.

12. a. Let $f(x)=x^4y^4$

$f(y)=y^4-y^4=0$

$\therefore (x-y)$ is a factor of x^4-y^4.

b. By division, the other factor is $x^3+x^2y+xy^2+y^3$.

$$
\begin{array}{r}
x^3+x^2y+xy^2+y^3 \\
x-y\,\overline{\big)\,x^4 \qquad\qquad\qquad -y^4} \\
\underline{x^4-x^3y} \qquad\qquad\qquad \\
x^3y \qquad\qquad\quad \\
\underline{x^3y-x^2y^2} \qquad\quad \\
x^2y^2 \qquad\quad \\
\underline{x^2y^2-xy^3} \quad \\
xy^3-y^4 \\
\underline{xy^3-y^4} \\
0
\end{array}
$$

c. From the pattern of **2. b**.

$x^4-81=x^4-(3)^4$

$=(x-3)\left(x^3+x^2(3)+x(3)^2+(3)^3\right)$

$=(x-3)(x^3+3x^2+9x+27)$

13. a. Let $f(x)=x^5-y^5$

$f(y)=y^5-y^5=0$

$\therefore (x-y)$ is a factor of x^5-y^5.

b. By dividing,

$$\begin{array}{r|l} & x^4 + x^3y + x^2y^2 + xy^3 + y^4 \\ \hline x-y & x^5 \qquad\qquad\qquad - y^5 \\ & \underline{x^5 - x^4y} \\ & \quad x^4y \\ & \quad \underline{x^4y - x^3y^2} \\ & \qquad x^3y^2 \\ & \qquad \underline{x^3y^2 - x^2y^3} \\ & \qquad\quad x^2y^3 \\ & \qquad\quad \underline{x^2y^3 - xy^4} \\ & \qquad\qquad xy^4 - y^5 \\ & \qquad\qquad \underline{xy^4 - y^5} \\ & \qquad\qquad\qquad 0 \end{array}$$

$x^5 - y^5 = (x-y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$

c.

$$\begin{aligned} x^5 - 32 &= x^5 - 2^5 \\ &= (x-2)\left(x^4 + x^3(-2) + x^2(-2)^2 + x(-2)^3 + (-2)^4\right) \\ &= (x-2)(x^4 - 2x^3 + 4x^2 - 8x + 16) \end{aligned}$$

14. a. Let $f(x) = x^n - y^n$.
Since $f(y) = y^n - y^n = 0$, then $(x-y)$ is a factor of $x^n - y^n$ by the Factor Theorem.

b. From the factoring pattern developed in questions 2 and 3, the other factor is
$x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \cdots xy^{n-2} + y^{n-1}$.

15. Let $f(x) = (x+a)^5 + (x+c)^5 + (a-c)^5$

$$\begin{aligned} f(-a) &= (a-a)^5 + (-a+c)^5 + (a-c)^5 \\ &= 0 + [(-1)(a-c)]^5 + (a-c)^5 \\ &= (-1)^5(a-c)^5 + (a-c)^5 \\ &= -(a-c)^5 + (a-c)^5 \\ &= 0 \end{aligned}$$

$\therefore (x+a)$ is a factor of $f(x)$.

16. Let $f(x) = x^3 - (a+b+c)x^2 + (ab+bc+ca)x - abc$.

$$\begin{aligned} f(a) &= a^3 - (a+b+c)a^2 + (ab+bc+ca)a - abc \\ &= a^3 - a^3 - a^2b - a^2c + a^2b + abc + a^2c - abc \\ &= 0 \end{aligned}$$

$\therefore (x-a)$ is a factor of $f(x)$.

17. If $n \in N, (x+y)$ will be a factor of $f(x) = x^n + y^n$ if and only if n is an odd number. If n is an odd number, then

$$\begin{aligned} f(-y) &= (-y)^n + y^n \\ &= y^n + y^n \\ &= 0. \end{aligned}$$

However, if n is an even number, then

$$\begin{aligned} (f(-y) &= (-y)^n + y^n \\ &= y^n + y^n \\ &\neq 0, \end{aligned}$$

and in order for $(x+y)$ to be a factor, $f(-y) = 0$.

18. Let $f(x) = x^5 + y^5$.

$$\begin{aligned} \text{Since } f(y) &= (-y)^5 + y^5 \\ &= -y^5 + y^5 \\ &= 0 \end{aligned}$$

then $(x+y)$ is a factor of $f(x)$.
By dividing,
$x^5 + y^5 = (x+y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$.

19. $f(x) = x^3 + 2x^2 + 5x + 12$
Since $f(x)$ is a cubic function, it could have at least one factor of the form $(x-p)$ where p is negative. Possible values for p are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6$, and ± 12. Using the graphing calculator, the function has no value for p. We cannot find a rational number for p.

Exercise 2.2

1. a. $f(x) = 2x^2 + 9x - 5$
For factors with integer coefficients, the first terms must be either $2x$ or x. Since the only factors of 5 are 5and 1, the possible values of $\frac{p}{q}$ are
$\pm\frac{1}{2}, \pm\frac{5}{2}, \pm 1$, and ± 5.

b. $f(x) = 3x^3 - 4x^2 + 7 + 8$

For factors, the first terms must be $3x$ and x, and the second terms must be $\pm 1, \pm 2, \pm 4$, and ± 8.

A graph of the function shows $k = \frac{p}{q}$ can be between 0 and –1. Since p divides 3 and q divides 8, we try $-\frac{1}{3}$, and $-\frac{2}{3}$.

c. $f(x) = 4x^3 + 3x^2 - 11x + 2$

The first terms of the possible factors must be $4x$, $2x$, or x. The second terms must be $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6$, or ± 12.

Graphing gives possible values for $\frac{p}{q}$ as between –2 and –3. Therefore, there are no possible values for $\frac{p}{q}$.

d. $f(x) = 8x^3 - 7x^2 + 23x - 4$

The first terms of the possible factors are $8x$, $4x$, $2x$, or x.

The second terms could be $\pm 1, \pm 2, \pm 3$, or ± 4.
By graphing, we see possible values for k are between 0 and 1, closer to 0.

Possible values for $\frac{p}{q}$ are then $\frac{1}{8}, \frac{1}{4}$, or $\frac{3}{8}$.

e. $f(x) = 6x^3 - 7x^2 + 4x + 3$

The first terms could be $6x, 3x, 2x$, or x. (q)

The second terms could be $\pm 1, \pm 2, \pm 3$, or ± 6. (p)

By graphing, we see possible values for k are between 0 and –1.

Possible values for $\frac{p}{q}$ are $-\frac{1}{3}$ and $-\frac{1}{2}$.

2. If $f\left(\frac{3}{2}\right) = 0$, then $(2x - 3)$ is a factor of $f(x)$. Since $(x - 2)$ is also a factor and $f(x)$ is cubic, then $f(x) = (x-2)(2x-3)(ax+b)$ where $a, b \in I, a \neq 0$.

But $f(4) = 50$

$$\begin{aligned} \therefore (4-2)(2(4)-3)(4a+b) &= 50 \\ 2(5)(4a+b) &= 50 \\ 4a + b &= 5 \\ \text{or} \quad b &= 5 - 4a. \end{aligned}$$

Since there are many values that satisfy this equation, we select one possibility, i.e., $a = 1$, $b = 1$. One possibility is $f(x) = (x-2)(2x-3)(x+1)$.

3. If $g(3) = 0$, then $(x - 3)$ is a factor of $g(x)$.

If $g\left(-\frac{3}{4}\right) = 0$, then $(4x + 3)$ is a factor.

Since $(x + 2)$ is a given factor as well, then the quartic function is $g(x) = (x+2)(x-3)(4x+3)(ax+b)$, where $a, b. \in I$ $a \neq$ 0.

$$\begin{aligned} \text{Since} \quad g(1) &= -84 \\ (1+2)(1-3)(4+3)(a+b) &= -84 \\ (3)(-2)(7)(a+b) &= -84 \\ a + b &= 2. \end{aligned}$$

Let $a = 1$, then $b = 1$.
The function is
$g(x) = (x+2)(x-3)(4x+3)(x+1)$.

4. **a.** $f(x) = 2x^3 + x^2 + x - 1$

From the graph and the possibilities for $\frac{p}{q}$, we see possible values for k is $\frac{1}{2}$.

Using the $\boxed{\text{CALC}}$ function, we have $x = \frac{1}{2}, y = 0$.

Therefore, $f\left(\frac{1}{2}\right)=0$, so $\left(x-\frac{1}{2}\right)$ or $(2x-1)$ is a factor.

$$\begin{aligned} f(x) &= 2x^3+x^2+x-1 \\ &= (2x-1)(x^2+kx+1), \text{ where } k\in I \\ &= 2x^3+(2k-1)x^2+\ldots \end{aligned}$$

Comparing coefficients, $2k-1=1$.

$$2k=2$$
$$k=1$$

$\therefore 2x^3+x^2+x-1=(2x-1)(x^2+x+1)$.

c. $f(x)=6x^3-17x^2+11x-2$

From the graph, we see possible values for k are between 0 and 1, and at 2. Checking the $\boxed{\text{CALC}}$ and $\boxed{\text{VALUE}}$ functions,

$f(2)=0$ and $f\left(\frac{1}{2}\right)=0$.

So, $(x-2)$ and $(2x-1)$ are factors of $f(x)$.

$\therefore 6x^3-17x^2+11x-2=(2x-1)(x-2)(3x-1)$.

e. $f(x)=5x^4+x^3-22x^2-4x+8$

From the graph, we see that there are four factors. Possible values for $k=\frac{p}{q}$ are –2, between –1 and 0, between 0 and 1, and 2. Testing, $f(-2)=0$ and $f(+2)=0$. So, $(x+2)$ and $(x-2)$ are factors.

$$\begin{aligned} &\therefore 5x^4+x^3-22x^2-4x+8 \\ &= (x+2)(x-2)(5x^2+kx-2) \\ &= (x^2-4)(5x^2+kx-2) \\ &= 5x^4+kx^3+\ldots \end{aligned}$$

Comparing coefficients, $k = 1$.

$\therefore 5x^4+x^3-22x^2-4x+8=(x+2)(x-2)(5x^2+x-2)$.

f. $f(x)=18x^3-15x^2-x+2$

From the graph, we see that there are three factors, one between –1 and 0, and two between 0 and 1. The first terms could be $1x, 2x, 3x, 6x, 9x,$ and $18x$. (q)

The second terms could be ±1, or ± 2. (p)

$(3x+1)$ may be a factor.

$$\begin{aligned} \text{Testing, } f\left(-\frac{1}{3}\right) &= 18\left(-\frac{1}{3}\right)^3-15\left(-\frac{1}{3}\right)^2-\left(-\frac{1}{3}\right)+2 \\ &= -\frac{18}{27}-\frac{15}{9}+\frac{1}{3}+2 \\ &= 0. \end{aligned}$$

$\therefore (3x+1)$ is a factor.

$$\begin{aligned} &\text{So, } 18x^3-15x^2-x+2 \\ &= (3x+1)(6x^2+kx+2) \\ &= 18x^3+(3k+6)x^2+\ldots \end{aligned}$$

Comparing coefficients, $3k+6=-15$

$$3k=-21$$
$$k=-7.$$

Therefore, $18x^3-15x^3-x+2=(3x+1)(6x^2-7x+2)$
$=(3x+1)(3x-2)(2x-1)$.

g. $f(x)=3x^4-5x^3-x^2-4x+4$

There are two possible values for $k=\frac{p}{q}$ at 2 and between 0 and 1. Using the $\boxed{\text{CALC}}$ function and testing $\boxed{\text{VALUE}}$ of $x=2$ and $\frac{2}{3}$, we find

$f(2)=0$ and $f\left(\frac{2}{3}\right)=0$.

So, the two factors are $(x-2)$ and $(3x-2)$.

$$\begin{aligned} &3x^4-5x^3-x^2-4x+4 \\ &= (x-2)(3x-2)(x^2+kx+1) \\ &= (3x^2-8x+4)(x^2+kx+1) \\ &= 3x^4+(3k-8)x^3+\ldots \end{aligned}$$

Comparing coefficients, $3k-8=-5$

$$3k=3$$
$$k=1.$$

$\therefore 3x^4-5x^3-x^2-4x+4=(x-2)(3x-2)(x^2+x+1)$.

5. **a.** $f(x) = px^3 + (p-q)x^2 + (-2p-q)x + 2q$

The first term can be p or 1.

The second term can be $\pm 2, \pm q,$ or $\pm 2q$.

We try $x = -2$:

$$\begin{aligned} f(-2) &= p(-2)^3 + (p-q)(-2)^2 + (-2p-q)(-2) + 2 \\ &= -8p + 4p - 4q + 4p + 2q + 2q \\ &= 0. \end{aligned}$$

So, $(x+2)$ is a factor of $f(x)$.

$$\begin{aligned} \therefore\ & px^3 + (p-q)x^2 + (-2p-q)x + 2q \\ &= (x+2)(px^2 + kx + q) \\ &= px^3 + (k+2p)x^2 + \ldots \end{aligned}$$

By comparing coefficients, we have

$$\begin{aligned} k + 2p &= p - q \\ k &= -p - q. \end{aligned}$$

$$\begin{aligned} \therefore\ & px^3 + (p-q)x^2 + (-2p-q)x + 2q \\ &= (x+2)(px^2 - (p+q)x + q). \end{aligned}$$

b. $f(x) = abx^3 + (a - 2b - ab)x^2 + (2b - a - 2)x + 2$

If the factors are integer values, the first term can be ax, bx, abx, or x and the second term can be ± 1 or ± 2.

So, $K = \dfrac{p}{q}$ can be $\pm 1,\ \pm 2,\ \pm \dfrac{1}{a},\ \pm \dfrac{1}{b},\ \pm \dfrac{1}{ab},$ etc.

$$\begin{aligned} f(1) &= ab + a - 2b - ab + 2b - a - 2 + 2 \\ &= 0 \end{aligned}$$

$\therefore (x-1)$ is a factor.

$$\begin{aligned} \text{So, }\ & abx^3 + (a - 2b - ab)x^2 + (2b - a - 2)x + 2 \\ &= (x-1)(abx^2 + kx - 2) \\ &= abx^3 + (k - ab)x^2 + (-2 - k)x + 2. \end{aligned}$$

Comparing coefficients, $k - ab = a - 2b - ab$

$$k = a - 2b.$$

$$\begin{aligned} \therefore\ & abx^3 + (a - 2b - ab)x^2 + (2b - a - 2)x + 2 \\ &= (x-1)(abx^2 + (a - 2b)x - 2) \\ &= (x-1)(ax-2)(bx+1). \end{aligned}$$

Exercise 2.3

3. **a.** Since the x-intercepts are –3, 0, and 2, $(x+3)$, x, and $(x-2)$ must be factors of the cubic function. Therefore, $f(x) = k(x)(x-2)(x+3)$, where k is a constant, represents the family of cubic functions.

b. If $(-1, 12)$ lies on the graph of one member of the family, then $(-1, 12)$ must satisfy the equation.

Substituting, $12 = k(-1)(-1-2)(-1+3)$

$$\begin{aligned} 12 &= 6k \\ k &= 2. \end{aligned}$$

So, the particular member is

$f(x) = 2x(x-2)(x+3).$

4. **a.** Since the x-intercepts are –2, –1, and 1, then $(x+2)$, $(x+1)$, and $(x-1)$ are factors of $f(x)$.

$\therefore f(x) = k(x-1)(x+1)(x+2)$, where k is a constant.

6. For roots $1, 2,$ and $\dfrac{3}{5}$, the factors must be $(x-1), (x-2),$ and $(5x-3)$. A polynomial equation with these roots is $(x-1)(x-2)(5x-3) = 0$.

7. If 2 is a root of the equation, substituting $x = 2$ will satisfy the equation. Then,

$$\begin{aligned} 2(2)^3 - 5k(2)^2 + 7(2) + 10 &= 0 \\ 16 - 20k + 14 + 10 &= 0 \\ -20k &= -40 \\ k &= 2. \end{aligned}$$

8. b. $x^2+2x+10=0$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(10)}}{2(1)}$$
$$=\frac{-2\pm 6i}{2}$$
$$=-1\pm 3i$$

e.
$$x^3=x$$
$$x^3-x=0$$
$$x(x^2-1)=0$$
$$x(x-1)(x+1)=0$$
$x=0$ or $x-1=0$ or $x+1=0$
$x=0$ or $x=1$ or $x=-1$

f.
$$x^4-1=0$$
$$(x^2+1)(x^2-1)=0$$
$x^2+1=0$ or $x^2-1=0$
$x^2=-1$ or $x^2=1$
$x=\pm i$ or $x=\pm 1$

h.
$$8x^3-27=0$$
$$(2x)^3-3^3=0$$
$$(2x-3)(4x^2+6x+9)=0$$
$2x-3=0$ or $4x^2+6x+9=0$

$2x=3$ or $x=\dfrac{-6\pm\sqrt{36-4(4)(9)}}{2(4)}$

$x=\dfrac{3}{2}$ or $x=\dfrac{-6\pm\sqrt{-108}}{8}$

$$=\frac{-6\pm 6i\sqrt{3}}{8}$$
$$=\frac{-3\pm 3i\sqrt{3}}{4}$$

i. Let $f(x)=x^3-3x^2-4x+12.$
$$f(2)=2^3-3(2)^2-4(2)+12$$
$$=0$$
$\therefore (x-2)$ is a factor of $f(x)$.

By dividing, the other factor is x^2-x-6.

$$\begin{array}{r} x^2-x-6 \\ x-2\,\overline{)\,x^3-3x^2-4x+12} \\ \underline{x^3-2x^2} \\ -x^2-4x \\ \underline{-x^2+2x} \\ -6x+12 \\ \underline{-6x+12} \\ 0 \end{array}$$

$$x^3-3x^2-4x+12=0$$
$$(x-2)(x^2-x-6)=0$$
$$(x-2)(x-3)(x+2)=0$$
$x-2=0$ or $x-3=0$ or $x+2=0$
$x=2$ or $x=3$ or $x=-2$

j. Let $f(x)=x^3-9x^2+26x-24.$
$$f(2)=2^3-9(2)^2+26(2)-24$$
$$=8-36+52-24$$
$$=0$$

$\therefore (x-2)$ is a factor of $f(x)$.

So, $x^3-9x^2+26x=24$
$$x^3-9x^2+26x-24=0$$
$$\left.\begin{array}{l}(x-2)(x^2-7x+12)=0\\(x-2)(x-4)(x-3)=0\end{array}\right\}\text{ by comparing coefficients or by division}$$
$x-2=0$ or $x-4=0$ or $x-3=0$
$x=2$ or $x=4$ or $x=3$

l. $\left.\begin{array}{l}\text{Let } f(x)=x^3-2x^2-15x+36\\ f(3)=27-2(9)-15(3)+36\end{array}\right\}$ To find the zeros, use this $\pm 1, \pm 2, \pm 3, \ldots$ all factors of 36.
$$=27-18-45+36$$
$$=0$$

$\therefore (x-3)$ is a factor of $f(x)$.
$$x^3-2x^2-15x+36=0$$
$(x-3)(x^2+x-12)=0$ by inspection
$$(x-3)(x+4)(x-3)=0$$
$x-3=0$ or $x+4=0$ or $x-3=0$
$x=3$ or $x=-4$ or $x=3$

Then $x=3$ or -4.

m. $x^3 + 8x + 10 = 7x^2$

$x^3 - 7x^2 + 8x + 10 = 0$

Let $f(x) = x^3 - 7x^2 + 8x + 10.$

To find the zeros, we try $\pm 1, \pm 2, \pm 5, \ldots$ all factors of 10.

$f(5) = 5^3 - 7(5)^2 + 8(5) + 10$
$= 0$

$\therefore (x-5)$ is a factor of $f(x)$.

So, $x^3 - 7x + 8x + 10 = 0$

$(x-5)(x^2 - 2x - 2) = 0$ by inspection

$x - 5 = 0$ or $x^2 - 2x - 2 = 0$

$x = 5$ or $x = \dfrac{2 \pm \sqrt{2^2 - 4(1)(-2)}}{2(1)}$

$= \dfrac{2 \pm \sqrt{12}}{2}$

$= \dfrac{2 \pm 2\sqrt{3}}{2}$

$= 1 \pm \sqrt{3}$

n. $x^3 - 3x^2 + 16 = 6x$

$x^3 - 3x^2 - 6x + 16 = 0$

To find x, such that $f(x) = 0$, we try the factors of 16, i.e., $\pm 1, \pm 2$, etc.

$f(2) = 2^3 - 3(2)^2 - 6(2) + 16$
$= 0$

$\therefore (x - 2)$ is a factor of $f(x)$.

$x^3 - 3x^2 - 6x + 16 = 0$

$(x-2)(x^2 - x - 8) = 0$

$x - 2 = 0$ or $x^2 - x - 8 = 0$

$x = 2$ or $x = \dfrac{1 \pm \sqrt{1 - 4(1)(-8)}}{2(1)}$

$= \dfrac{1 \pm \sqrt{33}}{2}$

9. b. $4x^3 + 19x^2 + 11x - 4 = 0$

By graphing $f(x) = 4x^3 + 19x^2 + 11x - 4,$ it appears that the x-intercepts are –4, –1, and 0.25.

By using $\boxed{\text{VALUE}}$ selection in $\boxed{\text{CALC}}$ mode, we see $f(-4) = f(-1) = f(0.25) = 0.$

$\therefore (x+4)$ and $(x+1)$ and $(4x+1)$ are factors of $f(x)$.

By taking the product, we can verify this.

$(x+4)(x+1)(4x+1)$
$= (x^2 + 5x + 4)(4x - 1)$
$= 4x^2 - x^2 + 20x^2 - 5x + 16x - 4$
$= 4x^2 + 19x^2 + 11x - 4$
$\therefore x = -4$ or -1 or 0.25.

d. $4x^4 - 2x^3 - 16x^2 + 8x = 0$

$x(4x^3 - 2x^2 - 16x + 8) = 0$

$x[2x^2(2x-1) - 8(2x-1)] = 0$

$x[(2x-1)(2x^2 - 8)] = 0$

$x = 0$ or $2x - 1 = 0$ or $2x^2 - 8 = 0$

or $x = \dfrac{1}{2}$ or $2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

f. $x^4 - 7 = 6x^2$

$x^4 - 6x^2 - 7 = 0$

$(x^2 - 7)(x^2 + 1) = 0$

$x^2 - 7 = 0$ or $x^2 + 1 = 0$

$x^2 = 7$ $\quad$ $x^2 = -1$

$x = \pm\sqrt{7}$ $\quad$ $x = \pm i$

h.
$$(x+1)(x+5)(x+3) = -3$$
$$(x^2+6x+5)(x+3) = -3$$
$$x^3+3x^2+6x^2+18x+5x+15 = -3$$
$$x^3+9x^2+23x+18 = 0$$

Let $f(x) = x^3+9x^2+23x+18\ldots$

Try $x = \pm 1, \pm 2, \pm 3$.

$$f(-2) = (-2)^3 + 9(-2)^2 + 23(-2) + 18$$
$$= -8+36-46+18 = 0$$

$\therefore (x+2)$ is a factor of $x+2$

By division,

$x^3+9x^2+23x+18 = (x+2)(x^2+7x+9) = 0$

$x+2=0$ or $x^2+7x+9=0$

$x=-2$ $\quad$ $x = \dfrac{-7\pm\sqrt{49-4(1)(9)}}{2(1)}$

$= \dfrac{-7\pm\sqrt{13}}{2}.$

10. a. $x^8-10x^4+9=0$

$(x^4-9)(x^4-1)=0$

$x^4-9=0$ or $x^4-1=0$

$(x^2-3)(x^2+3)=0$ or $(x^2-1)(x^2+1)=0$

$x^2-3=0$ or $x^2+3=0$ or $x^2-1=0$ or $x^2+1=0$

$x=\pm\sqrt{3}$ or $x=\pm i\sqrt{3}$ or $x=\pm 1$ or $x=\pm i$

b. $x^6-7x^3-8=0$

Let $x^3=a$.

$a^2-7a-8=0$

$(a-8)(a+1)=0$

$a-8=0$ or $a+1=0$

But $a=x^3$; substituting,

$x^3-8=0$ or $x^3+1=0$

$(x-2)(x^2+2x+4)=0$ or $(x+1)(x^2-x+1)=0$

$x-2$ or $x^2+2x+4=0$ or $x+1=0$ or $x^2-x+1=0$

$x=2$ or $x=\dfrac{-2\pm\sqrt{4-4(1)(4)}}{2(1)}$

or $x=-1$ or $x=\dfrac{1\pm\sqrt{1-4(1)(1)}}{2}$

$x=2$ or $x=\dfrac{-2\pm\sqrt{-12}}{2}$ or $x=-1$ or $x=\dfrac{1\pm\sqrt{-3}}{2}$

$x=2$ or $x=-1\pm 3i$ or $x=-1$ or $x=\dfrac{1\pm\sqrt{3}}{2}$

c. $(x^2-x)^2-8(x^2-x)+12=0$

Let $a=x^2-x.$

Then substituting,

$a^2-8a+12=0$

$(a-6)(a-2)=0$

$a-6=0$ or $a-2=0.$

But $a=x^2-x$

$\therefore x^2-x-6=0$ or $x^2-x-2=0$

$(x-3)(x+2)=0$ $\quad$ $(x-2)(x+1)=0$

$x-3=0$ or $x+2=0$ $\quad$ $x-2=0$ or $x+1=0$

$x=3$ or $x=-2$ $\quad$ $x=2$ or $x=-1.$

d. $\left(x-\dfrac{1}{x}\right)^2-\dfrac{77}{12}\left(x-\dfrac{1}{x}\right)+10=0$

Let $a=x-\dfrac{1}{x}.$

$a^2-\dfrac{77}{12}a+10=0$

$12a^2-77a+120=0$

Since there are so many possible integers to try, we use the quadratic formula.

$a=\dfrac{77\pm\sqrt{(-77)^2-4(12)(120)}}{2(12)}$

$=\dfrac{77\pm 13}{24}$

$=\dfrac{15}{4}$ or $\dfrac{8}{3}$

But $a=x-\dfrac{1}{x}$

$\therefore x-\dfrac{1}{x}=\dfrac{15}{4}$ or $x-\dfrac{1}{x}=\dfrac{8}{3}.$

Since $x\neq 0$

$x^2-1=\dfrac{15x}{4}$ or $3x^2-8x-3=0$

$4x^2-15x-4=0$

$(4x+1)(x-4)=0$ or $(3x+1)(x-3)=0$

$4x+1=0$ or $x-4=0$ or $3x+1=0$ or $x-3=0$

$x=-\dfrac{1}{4}$ or $x=4$ or $x=-\dfrac{1}{3}$ or $x=3.$

e. $(3x-5)(3x+1)^2(3x+7)+68=0$

Let $a=3x+1$.

Then $3x-5=a-6$ and $3x+7=a+6$.

Substituting,

$$(a-6)(a)^2(a+6)+68=0$$
$$a^2(a^2-36)+68=0$$
$$a^4-36a^2+68=0$$
$$(a^2-34)(a^2-2)=0$$
$$a^2-34=0 \text{ or } a^2-2=0.$$

But $a=3x+1$

$\therefore (3x+1)^2-34=0$ or $(3x+1)^2-2=0$

$$9x^2+6x-33=0 \qquad 9x^2+6x-1=0$$
$$3x^2+2x-11=0 \qquad x=\frac{-6\pm\sqrt{6^2-4(9)(-1)}}{2(9)}$$
$$x=\frac{-2\pm\sqrt{2^2-4(3)(-11)}}{2(3)} \qquad =\frac{-6\pm\sqrt{72}}{18}$$
$$x=\frac{-2\pm\sqrt{136}}{6} \qquad =\frac{-6\pm6\sqrt{2}}{18}$$
$$=\frac{-1\pm\sqrt{34}}{3} \qquad =\frac{-1\pm\sqrt{2}}{3}.$$

f. $(x^2+6x+6)(x^2+6x+8)=528$

Let $a=x^2+6x+6$.

Then substituting,

$$a(a+2)=528$$
$$a^2+2a-528=0$$
$$(a+24)(a-22)=0$$
$$a+24=0 \text{ or } a-22=0$$

But $a=x^2+6x+6$

$$\therefore x^2+6x=6+24=0 \text{ or } x^2=6x+6-22=0$$
$$x^2+6x+30=0 \qquad x^2+6x-16=0$$
$$x=\frac{-6\pm\sqrt{6^2-4(1)(30)}}{2(1)} \qquad (x+8)(x-2)=0$$
$$=\frac{-6\pm\sqrt{-84}}{2} \qquad x+8=0 \text{ or } x-2=0$$
$$=-3\pm i\sqrt{21} \qquad x=-8 \text{ or } x=2$$

11. The volume of ice is given by $y=8x^3+36x^2+54x$.

If the volume of ice is 2170 cm^3,

$$8x^3+36x^2+54x=2170$$
$$8x^3+36x^2+54x-2170=0$$
$$4x^3+18x^2+27x-1085=0.$$

By graphing $y=4x^3+18x^2+27x-1085$, we find $y=0$ when $x=5$.

Since x represents the thickness of ice that gives a specific volume, there is only one value, i.e., the thickness of ice is 5 cm.

12. b. $x^3-2x^2-8x+13=0$

Graphing $y=x^3-2x^2-8x+13$, we find the roots using CALC mode and ZERO, locating roots between –3 and –2, and 1, 2, 3 and 4. The roots are $x \doteq -2.714, 1.483$, and 3.231.

c. $2x^3-6x^2+4=0$

Graphing $y=2x^3-6x^2+4$, the roots lie between –1 and 0, between 2 and 3, and exactly 1. Using ZERO option in CALC mode, we find roots at –0.732 and 2.732.

$\therefore$ the roots are 1, –0.732, and 2.732.

13. Let the dimensions of the box have a height of x cm, a width of $(x + 1)$ cm, and a length of $(x + 2)$ cm. The volume of the rectangular box is $V_0=x(x+1)(x+2)$ where volume, V, is in cm^3. The new dimensions are $2x$, $x + 2$, and $x + 3$.

$\therefore$ the new volume is $V_1=2x(x+2)(x+3)$.

The increase in volume is

$$V_1-V_0=120$$
$$2x(x+2)(x+3)-x(x+1)(x+2)=120$$
$$2x(x^2+5x+6)-x(x^2+3x+2)=120$$
$$2x^3+10x^2+12x-x^3-3x^2-2x=120$$
$$x^3+7x^2+10x-120=0$$

Let $f(x)=x^3+7x^2+10x-120.$

Since $f(3)=3^3+7(3)^2+10(3)-120$
$=0.$

then $(x-3)$ is a factor.
By dividing, $x^3+7x^2+10x-120=0$
becomes $(x-3)(x^2+10x+40)=0$

$x-3=0$ or $x^2+10x+40=0$

$$x=3 \text{ or } x=\frac{-10\pm\sqrt{10^2-4(1)(40)}}{2}$$
$$=\frac{-10\pm\sqrt{-60}}{2}$$
$$=-5\pm i\sqrt{15}.$$

But $x>0, x\in R$ since it represents the height of the box; $\therefore x=3$

$\therefore$ the dimensions are 3 cm by 4 cm by 5 cm.

14. The volume of the silo is to be 2000 m^3. Let r cm be the radius of the main section.

$$V=\underbrace{\pi r^2 h}_{\text{(main section)}}+\underbrace{\frac{1}{2}\left[\frac{4}{3}\pi r^3\right]}_{\text{(roof volume)}}$$

$$V=10\pi r^2+\frac{2}{3}\pi r^3$$

But $V=2000$

$$\therefore \frac{2}{3}\pi r^3+10\pi r^2-2000=0$$

Graphing,

$$y=\frac{2}{3}\pi r^3+10\pi r^2-2000,$$

we find one real root at $x=3.6859$. Therefore, the radius should be about 3.69 m.

15. $a=0.6t+2$ (1)
$v=0.3t^2+2t+4$ (2)
$s=0.1t^3+t^2+4t$ (3)

where a is acceleration in km/s^2 v is the velocity in km/s and s is the displacement in km.
If the displacement is 25 km, then
$25=0.1t^3+t^2+4t$ where $t>0, t\in R$
or $0.1t^3+t^2+4t-25=0.$
Using the graph of $f(t)=0.1t^3+t^2+4t-25$, we find one real root at $t=3.100833$.
Therefore, after 3.1 the rocket will have travelled 25 km.

Section 2.4 Investigation

Equation	a	b	c	Roots of Roots	Sum	Product of Roots
$x^2 - 5x + 6 = 0$	1	–5	6	3, 2	5	6
$x^2 + 3x - 28 = 0$	1	3	–28	–7, 4	–3	–28
$3x^2 + 19x + 6 = 0$	3	19	6	$-\frac{1}{3}, -6$	$-\frac{19}{3}$	2
$x^2 - 4x + 1 = 0$	1	-4	1	$2 \pm \sqrt{3}$	4	1
$2x^2 - 17x + 2 = 0$	2	–17	2	$\frac{17 \pm \sqrt{273}}{4}$	$\frac{17}{2}$	1
$5x^2 + x + 2 = 0$	5	1	2	$\frac{-1 \pm i\sqrt{39}}{10}$	$-\frac{1}{5}$	$\frac{2}{5}$

1. The sum of the roots of a quadratic equation is the opposite of the coefficient of the linear term divided by the coefficient of the quadratic term, that is, $x_1 + x_2 = -\frac{b}{a}$.

2. The product of the roots of a quadratic equation is the quotient of the constant term divided by the coefficient of the quadratic term, that is, $(x_1)(x_2) = \frac{c}{a}$.

Exercise 2.4

2. The quadratic equation is

x^2 – (sum of the roots) × (product of the roots) = 0.

a. The equation is $x^2 - 3x + 7 = 0$.

b. The equation is $x^2 + 6x + 4 = 0$.

c. The equation is

$$x^2 - \frac{1}{5}x - \frac{2}{25} = 0$$

or $25x^2 - 5x - 2 = 0$.

d. The equation is

$$x^2 + \frac{13}{12}x + \frac{1}{4} = 0$$

or $12x^2 + 13x + 3 = 0$.

e. The equation is

$$x^2 + 11x - \frac{2}{3} = 0$$

or $3x^2 + 33x - 2 = 0$.

3. **b.** $x_1 + x_2 = -5 + 8$ and $x_1 x_2 = (-5)(8)$
$= 3$ $= -40$

The equation is $x^2 - 3x - 40 = 0$.

c. $x_1 + x_2 = 3 + \frac{1}{3}$ and $x_1 x_2 = (3)\left(\frac{1}{3}\right)$
$= \frac{10}{3}$ $= 1$

The equation is $x^2 - \frac{10}{3}x + 1 = 0$
$3x^2 - 10x + 3 = 0.$

e. $x_1 + x_2 = -\frac{4}{5} + \frac{3}{25}$ and $(x_1)(x_2) = \left(-\frac{4}{5}\right)\left(\frac{3}{25}\right)$
$= -\frac{17}{25}$ $= -\frac{12}{125}$

The equation is
$$x^2 + \frac{17}{25}x - \frac{12}{125} = 0$$
or $125x^2 + 85x - 12 = 0.$

f.
$x_1 + x_2 = (2+i)(2-i)$ and $(x_1)(x_2) = (2+i)(2-i)$
$= 4$ $= 4 - i^2$
$= 5$

The equation is $x^2 - 4x + 5 = 0$.

4. *Solution 1*

Since 5 is a root of $2x^2 + kx - 20 = 0$, it must satisfy the equation. Therefore,
$$2(5)^2 + k(5) - 20 = 0$$
$$50 + k(5) - 20 = 0$$
$$5k = -30$$
$$k = -6.$$

Solution 2

Let h represent the second root of $2x^2 + kx - 20 = 0$.

The sum of the roots is $h + 5 = -\frac{k}{2}$ (1)

and the product is $5h = -\frac{20}{2}$
$5h = -10$
$h = -2$ (2)

Substituting into (1) $-2 + 5 = -\frac{k}{2}$
$3 = -\frac{k}{2}$
$k = -6$.

5. Let h represent the other root of $x^2 + x - 2k = 0$.
The sum of the roots is $h - 7 = -1$ or $h = 6$.
The product of the roots is $(-7)(h) = -2k$.
But $h = 6$,
$\therefore (-7)(6) = -2k$
$-42 = -2k$
$k = 21.$
The other root is 6, and $k = 21$.

6. Let x_1 and x_2 represent the roots of the given equations,
$x^2 + 8x - 1 = 0.$
$\therefore x_1 + x_2 = -8$ and $(x_1)(x_2) = -1.$
The roots of the required equation are $x_1 + 6$ and $x_2 + 6$.
For the sum of the new equation, the sum of the roots is
$(x_1 + 6) + (x_2 + 6) = x_1 + x_2 + 12.$
But $x_1 + x_2 = -8.$

Therefore, the sum of the roots of the new equation is $-8 + 12$ or 4.

For the new equation, the product of the roots is
$(x_1 + 6)(x_2 + 6)$
$= x_1x_2 + 6x_1 + 6x_2 + 36$
$= x_1x_2 + 6(x_1 + x_2) + 36$
$= (-1) + 6(-8) + 36$
$= -13.$

So, the new equation is $x^2 - 4x - 13 = 0$.

7. Let x_1 and x_2 represent the roots of the given equation.

$x_1 + x_2 = \frac{17}{2}$ and $(x_1x_2) = 1$

The roots of the required equation are $x_1 + 5$ and $x_2 + 5$.

For the new equation, the sum of the roots is

$$\begin{aligned} &(x_1 + 5) + (x_2 + 5) \\ &= (x_1 + x_2) + 10 \\ &= \frac{17}{2} + 10 \\ &= \frac{37}{2}. \end{aligned}$$

For the new equation, the product of the roots is

$$\begin{aligned} &(x_1 + 5)(x_2 + 5) \\ &= x_1x_2 + 5(x_1 + x_2) + 25 \\ &= 1 + 5\left(\frac{17}{2}\right) + 25 \\ &= \frac{137}{2}. \end{aligned}$$

So, the new equation is $x^2 - \frac{37}{2}x + \frac{237}{2} = 0$

or $\qquad 2x^2 - 37x + 137 = 0$.

8. Let x_1 and x_2 be the roots of $3x^2 + 7x + 3 = 0$,

$x_1 + x_2 = -\frac{7}{3}$ and $(x_1xx_2) = 1$. The roots of the required equation are $3x_1$ and $3x_2$. For the new equation, the sum of the roots is

$$\begin{aligned} &3x_1 + 3x_2 \\ &= 3(x_1 + x_2) \\ &= 3\left(-\frac{7}{3}\right) \\ &= -7. \end{aligned}$$

For the new equation, the product of the roots is

$$\begin{aligned} &(3x_1)(3x_2) \\ &= 9x_1x_2 \\ &= 9(1) \\ &= 9. \end{aligned}$$

Therefore, the new equation is $x^2 + 7x + 9 = 0$.

9. Let the roots of $4x^2 - 9x - 2 = 0$ be represented by x_1 and x_2.

$x_1 + x_2 = \frac{9}{4}$ and $x_1x_2 = -\frac{2}{4} = -\frac{1}{2}$

The roots of the required equation are x_1^2 and x_2^2.

For the new equation, the sum of the roots is $x_1^2 + x_2^2$.

But $\quad (x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$

$$\begin{aligned} \left(\frac{9}{4}\right)^2 &= (x_1^2 + x_2^2) + 2(x_1x_2) \\ \frac{81}{16} &= (x_1^2 + x_2^2) + 2\left(-\frac{1}{2}\right) \\ \therefore x_1^2 + x_2^2 &= \frac{81}{16} + 1 \\ &= \frac{97}{16} \end{aligned}$$

For the new equation, the product of the roots is

$$\begin{aligned} x_1^2x_2^2 &= (x_1x_2)^2 \\ &= \left(-\frac{1}{2}\right)^2 \\ &= \frac{1}{4}. \end{aligned}$$

So, the required equation is

$$x^2 - (x_1^2 + x_2^2)x + x_1^2x_2^2 = 0$$

$$x^2 - \frac{97}{16}x + \frac{1}{4} = 0$$

or $\qquad 16x^2 - 97x + 4 = 0$.

10. Let x_1 and x_2 be the roots of $5x^2 + 10x + 1 = 0$.

Then $x_1 + x_2 = -\frac{10}{5} = -2$ and $x_1x_2 = \frac{1}{5}$.

Since the roots of the required equation are their reciprocals, and the new roots are $\frac{1}{x_1}$ and $\frac{1}{x_2}$.

The sum of the new roots is $\frac{1}{x_1}+\frac{1}{x_2}$

$$=\frac{x_2+x_1}{x_1x_2}$$
$$=-\frac{2}{\frac{1}{5}}$$
$$=-10.$$

The product of the new roots is $\left(\frac{1}{x_1}\right)\left(\frac{1}{x_2}\right)$

$$=\frac{1}{\frac{1}{5}}$$
$$=5.$$

So, the required equation is $x^2+10x+5=0$.

11. Let the roots of the given equation be x_1 and x_2.

For $x^2+6x-2=0,\ x_1+x_2=-6$ and $x_1x_2=-2$.

The roots of the required equation are $\left(\frac{1}{x_1}\right)^2$ and $\left(\frac{1}{x_2}\right)^2$.

The sum of the new roots is $\frac{1}{x_1^2}+\frac{1}{x_2^2}$

$$=\frac{x_2^2+x_1^2}{x_1^2x_2^2}$$
$$=\frac{x_1^2+x_2^2}{(x_1x_2)^2}.$$

Now, $(x_1+x_2)^2=x_1^2+2x+x_2+x_2^2$

$(-6)^2=x_1^2+2(-2)+x_2^2.$

So, $x_1^2+x_2^2=36+4$

$=40.$

and $x_1^2x_2^2=-2$

so, $(x_1x_2)^2=4.$

Therefore, the sum of the new roots is

$$\frac{x_1^2+x_2^2}{(x_1x_2)^2}=\frac{40}{4}=10.$$

The product of the new roots is $\left(\frac{1}{x_1}\right)^2\left(\frac{1}{x_2}\right)^2$

$$=\frac{1}{x_1^2x_2^2}$$
$$=\frac{1}{(x_1x_2)^2}$$
$$=\frac{1}{(-2)^2}$$
$$=\frac{1}{4}.$$

The required equation is

x^2- (sum of roots)x + (product of roots) = 0.

$$x^2-10x+\frac{1}{4}=0$$
$$4x^2-40x+1=0$$

12. Let x_1 and x_2 be the roots of $2x^2+4x+1=0$.

$x_1+x_2=-\frac{4}{2}$ and $(x_1)(x_2)=\frac{1}{2}$

$=-2$

The roots of the new equation are x_1^3 and x_2^3.

The product of the new roots is $x_1^3x_2^3=(x_1x_2)^3$

$$=\left(\frac{1}{2}\right)^3$$
$$=\frac{1}{8}.$$

The sum of the new roots is $x_1^3+x_2^3$.

But $(x_1+x_2)^3=x_1^3+3x_1^2x_2+3x_1x_2^2+x_2^3$

$$(-2)^3=x_1^3+3\left(x_1^2x_2+3x_1x_2^2\right)+x_2^3$$
$$-8=x_1^3+x_2^3+3x_1x_2(x_1+x_2)$$
$$-8=x_1^3+x_2+^3+3\left(\frac{1}{2}\right)(-2).$$

So, $x_1^3+x_2^3=-5.$

Therefore, the new equation is $x^2+5x+\frac{1}{8}=0$

$$8x^2+40x+1=0.$$

13. A cubic equation with roots x_1, x_2, x_3 may be written as

$$(x-x_1)(x-x_2)(x-x_3)=0 \qquad (1)$$

Expanding, $\quad (x-x_1)(x^2-(x_2+x_3)x+x_2x_3)=0$

$$x^3-(x_2+x_3)x^2+x_2x_3x$$
$$-(x_1)x^2+(x_1x_2+x_1x_3)x-x_1x_2x_3=0$$
$$x^3-(x_1+x_2+x_3)x^2+(x_1x_2+x_2x_3+x_1x_3)x-x_1x_2x_3=0$$

Comparing the coefficients of this expanded expression with the general cubic equation

$$ax^3+bx^2+cx+d=0, \qquad (2)$$

we note that the cubic term must be 1 in order to compare these equations, therefore, dividing (2) by *a,* we get

$$x^3+\frac{b}{a}x^2+\frac{c}{a}x+\frac{d}{a}=0\,.$$

Now, $\quad x_1+x_2+x_3=-\dfrac{b}{a}$

$$x_1x_2+x_1x_3+x_2x_3=\frac{c}{a}$$

and $\quad x_1x_2x_3=-\dfrac{d}{a}.$

14. Given the roots of a cubic equation are $\frac{1}{2}$, 2, and 4,

$$x_1+x_2+x_3=\frac{1}{2}+2+4$$
$$=\frac{13}{2}$$

$$x_1x_2+x_1x_3+x_2x_3=\left(\frac{1}{2}\right)(2)+\left(\frac{1}{2}\right)(4)+(2)(4)$$
$$=11$$
$$x_1x_2x_3=\left(\frac{1}{2}\right)(2)(4)$$
$$=4.$$

Therefore, the cubic equation is

$$x^3-\frac{13}{2}x^2+11x-4=0$$

or $\quad 2x^3-13x^2+22x-8=0.$

15. Let the roots of $x^3-4x^2-2=0$ be represented by x_1, x_2, x_3. From the solution to question 13,

$$x_1+x_2+x_3=\frac{4}{1}=4$$

$$x_1x_2+x_1x_3+x_2x_3=3$$
$$x_1x_2x_3=3.$$

For the required equation, the roots are x_1+2, x_2+2, and x_3+2.

(i) $(x_1+2)+(x_2+2)+(x_3+2)=(x_1+x_2+x_3)+6$
$$=4+6$$
$$=10$$

(ii) $(x_1+2)(x_2+2)(x_1+2)(x_3+2)(x_2+2)\,(x_3+2)$

$$=[x_1x_2+2(x_1+x_2)+4]+[x_1x_3+2(x_3+x_1)+4]$$
$$+[x_2x_3+2(x_3+x_2)+4]$$

$$=x_1x_2+x_1x_3+x_2x_3+2$$
$$(x_1+x_2+x_3+x_1+x_3+x_2)+4+4+4$$

$$=(x_1x_2+x_1x_3+x_2x_3)+4(x_1+x_2+x_3)+12$$
$$=3+4(4)+12$$
$$=31$$

(iii) $(x_1+2)(x_2+2)(x_3+2)$

$$=(x_1+2)(x_2x_3+2(x_3+x_2)+4)$$
$$=x_1x_2x_3+2(x_1x_3+x_1x_2)+4(x_1+2x_2x_3+4x_3+4x_2)+8$$
$$=x_1x_2x_3+2(x_1x_2+x_1x_3+x_2x_3)+4(x_1+x_2+x_3)+8$$
$$=2+2(3)+4(4)+8$$
$$=32$$

The required equation is $x^3-10x^2+31x-32=0.$

16. A quartic equation with roots x_1, x_2, x_3, and x_4 may be written as $(x-x_1)(x-x_2)(x-x_3)(x-x_4)=0$.

Expanding, we have

$$\left(x^2-(x_1+x_2)x+x_1x_2\right)\left(x^2-(x_3+x_4)x+x_3x_4\right)=0$$

$$x^4-(x_3+x_4)x^3+x_3x_4x^2-(x_1+x_2)x^3-(x_1+x_2)(x_3+x_4)x^2-(x_1+x_2)x_3x_4x+(x_1x_2)x^2-(x_3+x_4)x_1x_2x+x_1x_2x_3x_4=0$$

$$x^4-(x_1+x_2+x_3+x_4)x^3+(x_1x_2+x_1x_3+x_1x_4+x_2x_3+x_2x_4+x_3x_4)x^2-(x_1x_3x_4+x_2x_3x_4+x_1x_2x_3+x_1x_2x_4)x+x_1x_2x_3x_4x=0$$

Comparing coefficients with the general quartic equation of $ax^4+bx^3+cx^2+dx+e=0$

or $x^4+\frac{b}{a}x^3+\frac{c}{a}x^2+\frac{d}{a}x+\frac{e}{a}=0$.

We have

$$x_1+x_2+x_3+x_4=-\frac{b}{a}$$

$$x_1x_2+x_1x_3+x_1x_4+x_2x_3+x_2x_4+x_3x_4=\frac{c}{a}$$

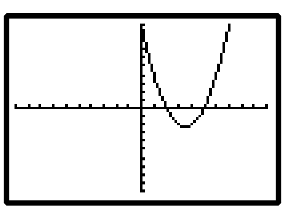
$$x_1x_2x_3+x_1x_3x_4+x_2x_3x_4+x_1x_2x_4=-\frac{d}{a}$$

$$x_1x_2x_3x_4=\frac{e}{a}.$$

Exercise 2.5

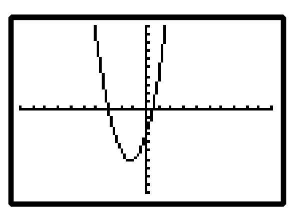
2. c. $x^2-7x+10\le 0$

From the graph of $y=x^2-7x+10$, it appears that $y = 0$ if $x = 2$ or 5. By substituting into the function, we see $y=0$ if $x=2$ or 5. So, the intercepts are 2 and 5. For $x^2-7x+10\le 0$, the graph is below or on the x-axis. Therefore, the solution is $2\le x\le 5$.



d. $2x^2+5x-3>0$

From the graph of $f(x)=2x^2+5x-3$, it appears that the intercepts are –3 and 0.5. Using the $\boxed{\text{VALUE}}$ mode in the $\boxed{\text{CALC}}$ function or by substituting, we find $f(-3)=f(0.5)=0$. The solution to $2x^2+5x-3>0$ is the set of values for x for which $f(x)$ is above the x-axis, i.e., $x<-3$ or $x>0.5$.

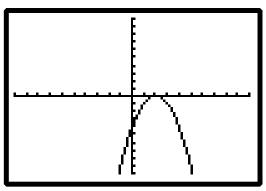


e. $-x^2 + 4x - 4 \geq 0$

For $y = f(x) = -x^2 + 4x - 4$, the intercept appears to be 2.

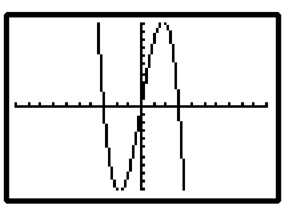
$f(2) = 0$

So, the solution to $-x + 4x - 4 \geq 0$ is the set of values for x where y is on or above the x-axis. But there is only one point that satisfies the condition, (2, 0), so the solution is $x = 2$.



f. $-x^3 + 9x \geq 0$

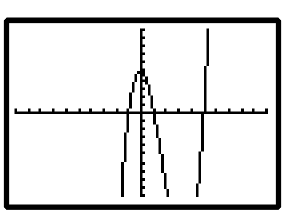
From the graph of $y = f(x) = -x^3 + 9x$, it appears the x-intercepts are –3, 0, and 3. Verifying this from $f(-3) = f(0) = f(3) = 0$, then the solution to $-x^3 + 9x \geq 0$ is the set of values for x where y is on or above the x-axis, i.e., $x \leq -3$ or $0 \leq x \leq 3$.



g. $x^3 - 5x^2 < x - 5$

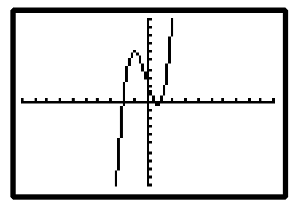
$x^3 - 5x^2 - x + 5 < 0$

The graph of $f(x) = x^3 - 5x^2 - x + 5$ is shown. We can verify intercepts at –1, 1, and 5 by using substitution or the $\boxed{\text{CALC}}$ function in $\boxed{\text{VALUE}}$ mode. The solution of $x^3 - 5x^2 - x + 5 < 0$ is the set of values for which $f(x)$ is below the x-axis, i.e., $x \leq -1$ or $1 \leq x \leq 5$.



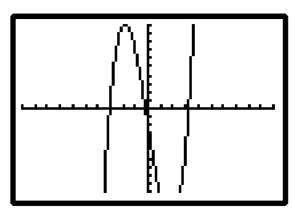
h. $2x^3 + x^2 - 5x + 2 \leq 0$

The graph of $f(x) = 2x^3 + x^2 - 5x + 2$ is shown. The intercepts appear to be –2, and between 0 and 1. By using the $\boxed{\text{CALC}}$ function and $\boxed{\text{VALUE}}$ and $\boxed{\text{ZERO}}$ modes, we find intercepts at –2, 0.5, and 1. The solution to $2x^3 + x^2 - 5x + 2 \leq 0$ is the set of values for x for which $f(x)$ is on or below the x-axis. The solution is $x \leq -2$ or $0.5 \leq x \leq 1$.



i. $x^3 - 10x - 2 \geq 0$

The graph of $f(x) = x^3 - 10x - 2$ is shown. The intercepts appear to be close to –3, 0, and 3. Using the $\boxed{\text{ZERO}}$ mode of the $\boxed{\text{CALC}}$ function, we find approximate x-intercepts at $x = -3.057$, –0.201, and 3.258. The solution will be those values for x for which $f(x)$ is on or above the x-axis. Then, for accuracy to one decimal place, the solution is $-3.1 \leq x \leq -0.2$ or $x \geq 3.3$.



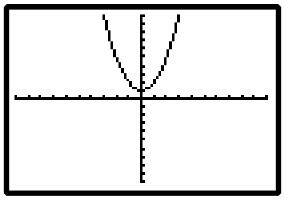
j. $x^2 + 1 > 0$

Solution 1

For all real values of x, $x^2 \geq 0$, so $x^2 + 1 \geq 0$. The solution is $x \in R$.

Solution 2

The graph of $f(x) = x^2 + 1$ shows all is above the x-axis. Therefore, the solution is R.



3. $v = -t^3 + 9t^2 - 27t + 21$

b. The intercept of the graph $v = -t^3 + 9t^2 - 27t + 21$ can be found to be $x \doteq 1.183$. For $v > 0$, $t < 1.183$ or t less than 60.3ºC.

c. Using the $\boxed{\text{TRACE}}$ function to find values for t that give values of v close to –20, and further refining the answer using the $\boxed{\text{VALUE}}$ mode in the $\boxed{\text{CALC}}$ function, we find $t \doteq 5.45$ when $v \doteq -20$. So, for $v < -20$, $t > 5.45$. The value of t is greater than $5.45(50^\circ)$ or 272.5ºC.

4. Graph $f(t) = 30t - 4.9t^2$ on your graphing calculator. Use the $\boxed{\text{TRACE}}$ function to find values for t that give values of h = 40. Using the $\boxed{\text{VALUE}}$ function in $\boxed{\text{CALC}}$ modes, we can refine our answers to give answers closer to 40. The projectile will be above 40 m between 2.0 s and 4.1 s after it is shot upwards.

5. Let the width of the base be x cm. The length is $2x$ cm, and the height is h cm. The total amount of wire is

$$\begin{aligned} 4(x) + 4(2x) + 4(h) &= 40 \\ 4x + 8x + 4h &= 40 \\ h &= \frac{40 - 12x}{4} \\ &= 10 - 3x. \end{aligned}$$

The volume of the solid is

$$\begin{aligned} v &= (x)(2x)(h) \\ &= (x)(2x)(10 - 3x) \\ &= -6x^3 + 20x^2. \end{aligned}$$

Graphing, $v = f(x) = -6x^3 + 20x^2$.

Use the $\boxed{\text{TRACE}}$ function to find values for x that give values of $f(x)$ to be close to 2 and 4.

$x \doteq 0.34, v \doteq 2.08$
$x \doteq 0.51, v \doteq 4.42$
and $x \doteq 3.23, v \doteq 4.5$
$x \doteq 3.32, v \doteq 2.1.$

We can use the $\boxed{\text{VALUE}}$ mode in the $\boxed{\text{CALC}}$ function to find closer approximations. We investigate the larger values, since the total amount of wire is 40 cm.

When $x = 3.30, v = 2.18$
and $x = 3.27, v = 4.06.$

So, to have the solid with a volume of approximately between 2 cm^3 and 4 cm^3, the width of the box must be between 3.27 cm and 3.3 cm.

Exercise 2.6

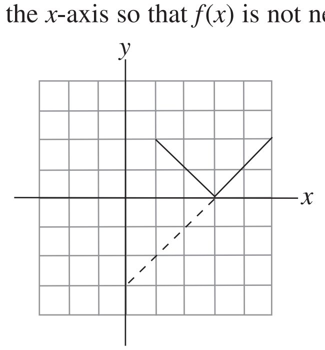
1. a. $|-3-7| = |-10|$
$= 10$

c. $|3| - |-5| + |3-9|$
$= 3 - (5) + |-6|$
$= 3 - 5 + 6$
$= 4$

d. $|9-3| + 5|-3| - 3|7-12|$
$= |6| + 5(3) - 3|-5|$
$= 6 + 15 - 3(5)$
$= 6$

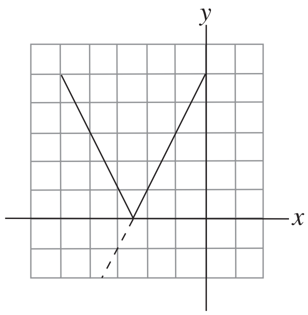
3. a. $f(x) = |x-3|,\ x \in R$

First, graph the line $f(x) = x - 3$. Then, reflect the portion of the graph that is below the x-axis in the x-axis so that $f(x)$ is not negative.



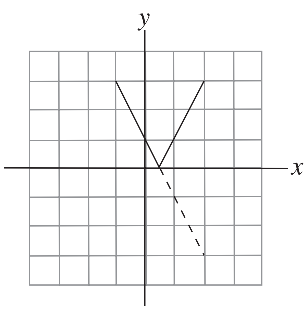
c. $h(x) = |2x + 5|$

Graph $h(x) = 2x + 5$. Then, reflect that portion of the graph below the x-axis in the x-axis.



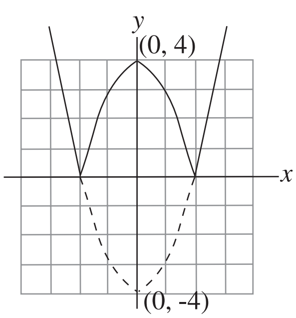
f. $g(x) = |1 - 2x|$

Graph $g(x) = 1 - 2x$. For the portion of the graph below the x-axis, reflect each point in the x-axis.



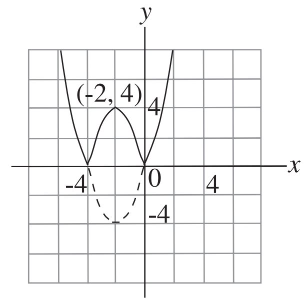
4. a. $y = |x^2 - 4|$

Graph the parabola $y = x^2 - 4$. Then, reflect the portion of the graph that is below the x-axis in the x-axis.



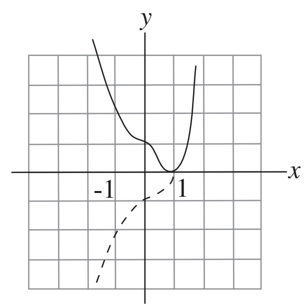
d. $y = |x^2 + 4x|$

First, graph the parabola $y = x^2 + 4x$. Then, reflect the portion of the graph where y is negative in the x-axis.

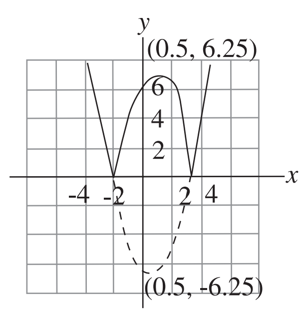


e. $y = |x^3 - 1|$

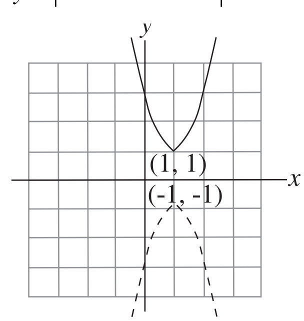
First, graph the cubic $y = x^3 - 1$. Then, reflect the portion of the graph where y is negative in the x-axis.



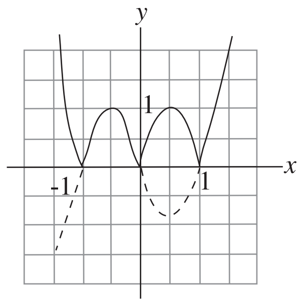
6. a. $y = |x^2 - x - 6|$



b. $y = |-2x^2 + 4x - 3|$



c. $y = |x^3 - x|$



7. **a.** $|2x-1| = 7$

Since $(2x-1)$ is 7 units from the origin,

either $2x-1=7$ or $2x-1=-7$

$2x = 8$ or $2x = -6$

$x = 4$ or $x = -3.$

b. $|3x+2| = 6$

Since $3x+2$ is 6 units from the origin,

either $3x+2=6$ or $3x+2=-6$

$3x = 4$ or $3x = -8$

$x = \frac{4}{3}$ or $x = -\frac{8}{3}.$

c. $|x-3| \le 9$

If $|x-3| \le 9$, then $(x-3)$ lies between –9 and 9 on the number line:

$$-9 \le x-3 \le 9.$$

Add 3: $-6 \le x \le 12.$

d. $|x+4| \ge 5$

$(x+4)$ lies beyond 5 and –5 on the number line,

so either $x+4 \ge 5$ or $x+4 \le -5$

$x \ge 1$ or $x \le -9.$

e. $|2x-3| < 4$

Then $-4 < 2x-3 < 4.$

Add 3: $-1 < 2x < 7$

Divide by 2: $-\frac{1}{2} < x < \frac{7}{2}$

f. $|x| = -5$

Since $|x|$ is always a positive number, there is no value of x for which $|x|$. Therefore, there is no solution.

8. **a.** $|x| = 3x+4$

By definition $|x| = \begin{cases} x \text{ if } x \ge 0 \\ -x \text{ if } x < 0 \end{cases}$

Therefore, if $x \ge 0$,

then $x = 3x+4$

$-2x = 4$

$x = -2,$

but only if $x \ge 0$, $\therefore x \ne -2$,

and if $x < 0$,

then $-x = 3x+4$

$-4x = 4$

$x = -1.$

Therefore, the solution is $x = -1$.

b. $|x-5| = 4x+1$

Solution 1

By definition, if $x-5 \ge 0$, then $x-5 = 4x+1$

if $x \ge 5$ $\quad -3x = 6$

$x = -2.$

But $x \ge 5$, $\therefore x \ne -2.$

Also, if $x-3 < 0$, then $-(x-5) = 4x+1$

$-5x = -4$

$x = \frac{4}{5}$ or 0.8.

Solution 2

First graph $y = |x-5|$

and $y = 4x+1.$

The point of intersection is $(0.8, 4.2)$

so $|x-5| = 4x+1$ when $x = 0.8.$

c. $|4x-8|=2x$

If $4x-8 \geq 0$, then $4x-8=2x$
$4x \geq 8$ $\quad$ $2x=8$
$x \geq 2$ $\quad$ $x=4$

If $4x-8<0$, then $4x-8=-2x$
$4x<8$ $\quad$ $6x=8$
$x<2$ $\quad$ $x=\frac{8}{6}=\frac{4}{3}$.

The solution $x=4$ or $x=\frac{4}{3}$ can be verified by graphing $y=|4x-8|$ and $y=2x$ and checking that the points of intersection occur when $x=4$, $x=\frac{4}{3}$.

d. $|x-1|<x$

Solution 1

Graphing $y_1=|x-1|$ and $y_2=x$ yields the following angle. We need to find the values for x for which $y_1<y_2$. Since the point of intersection is $\left(\frac{1}{2},\frac{1}{2}\right)$, $y_1<y_2$ when $x>\frac{1}{2}$.

Solution 2

Consider $|x-1|=x$
Then either $x-1=x$ or $x-1=-x$
$0x=4$ or $2x=1$
No solution. $\quad$ $x=\frac{1}{2}$.

Since $|x-1|=x$ when $x=\frac{1}{2}$, we test points on either side of $\frac{1}{2}$ on the number line to find for what values of x does $|x-1|<x$.

Test: $x=0$
L.S. $=|0-1|$ R.S. $=0$
$=1$
Since L.S. $\not<$ R.S., $x \neq 0$

Test: $x=1$
L.S. $=|1-1|$ R.S. $=1$
$=0$
Since L.S. $<$ R.S. $x=1$.

Therefore, the solution set is $\left\{x \mid x>\frac{1}{2}\right\}$.

Solution 3

By definition of absolute value,
if $x-1 \geq 0$, then $x-1<x$
$x \geq 1$ $\quad$ $0x<1$.

This is true for all x, $x \in R$.

And if $x-1<0$, then $-x+1<x$
$x<1$ $\quad$ $-2x<-1$
$x>\frac{1}{2}$.

Therefore, the solution set is $\left\{x \mid x>\frac{1}{2}\right\}$.

e. $|2x+4| \geq 12x$

Consider $|2x+4|=12x$.

Then, either $2x+4=12x$ or $2x+4=-12x$
$-10x=-4$ or $14x=-4$
$x=\frac{4}{10}=\frac{2}{5}$ or $x=-\frac{4}{14}=-\frac{2}{7}$.

But substituting each value into the equation gives only one solution, that is $x=\frac{2}{5}$.

Test values for x on either side of $x=\frac{2}{5}$.

Let $x=0$
L.S. $=|2(0)+4|$
$=4$
R.S. $=12(0)$
$=0$
Since L.S. $>$ R.S., $x=0$

Let $x=1$
L.S. $=|2(1)+4$
$=6$
R.S. $=12(1)$
$=12$
Since L.S. $\not>$ R.S, $x \neq 1$.

Therefore, the solution set is $\left\{x \mid x \leq \frac{2}{5}\right\}$.

This solution can be verified by graphing $y_1=|2x+4|$ and $y_2=|12x|$ and noting that $y_1 \geq y_2$ when $x \leq \frac{2}{5}$.

f. $|3x-1| \le 5|3x-1| - 16$

Consider $|3x-1| = 5|3x-1| - 16.$

Either $(3x-1) = 5(3x-1) - 16$ or $-(3x-1) = 5(-3x+1) - 16$

$3x - 1 = 15x - 5 - 16$ $\quad$ $-3x + 1 = -15x + 5 - 16$

$-12x = -20$ $\quad$ $12x = -12$

$x = \frac{20}{12} = \frac{5}{x}$ $\quad$ $x = -1.$

Both answers verify when substituted into the equation. Now, to find which values satisfy the inequality, we can use test values between and beyond -1 and $\frac{5}{3}$.

Test: $x = 0$

L.S. $= |3(0) - 1|$ R.S. $= |3(0) - 1| - 16$

$= 1$ $\quad$ $= -11$

Since L.S. $\not\le$ R.S. , then $x = 0$

Test: $x = -2$

L.S. $= |3(-2) - 1|$ R.S. $= 5|3(-2) - 1| - 16$

$= 7$ $\quad$ $= 19$

Since L.S. > R.S., then $x = -2$.

Test: $x = 2$

L.S. $= |3(2) - 1|$ R.S. $= 5|3(2) - 1| - 16$

$= 5$ $\quad$ $= 9$

Since L.S. < R.S., then $x = 2$.

So, the solution set is $\left\{ x | x \le -1 \text{ or } x \ge \frac{5}{3} \right\}$.

Or,

we can graph $y_1 = |3x-1|$ and $y_2 = 5|3x-1| - 16$ and, using the values of x found earlier, locate those values of x for which $y_1 \le y_2$.

g. $|x-2| + |x| = 6$

Solution 1

Graph $y_1 = |x-2| + |x|$ and $y_2 = 6$. The points of intersection are the points where $y_1 = y_2$, $\therefore x = -2$ or 4.

Solution 2

Since we need to concern ourselves when $|f(x)| = f(x)$ or $-f(x)$, we use the cases where $x < 0$, $0 < x < 2$, and $x > 2$.

Case 1:

If $x < 0$

then $|x-2| + |x| = 6$

becomes $-x + 2 - x = 6$

$-2x = 4$

$x = 2.$

$\therefore x = -2$ or $x = 4$

Case 2:

If $0 < x < 2$

then $|x-2| + |x| = 6$

becomes $-x + 2 + x = 6$

$0x = 4$

No solution.

Case 3:

If $x > 0$,

then $|x-2| + |x| = 6$

becomes $x - 2 + x = 6$

$2x = 8$

$x = 4.$

h. $|x+4| - |x-1| = 3$

Graph $y_1 = |x+4| - |x-1|$ and $y_2 = 3$.

Since the point of intersection is (0, 3), $y_1 = y_2$ when $x = 0$. Therefore, the solution is $x = 0$.

9. Since $|x - |x||$ is always positive, then $\frac{|x-|x||}{x}$ is positive when $x > 0$.

Since $x > 0$, $|x| \ge 0$

$\therefore |x - |x||$

$= |x - x|$

$= 0.$

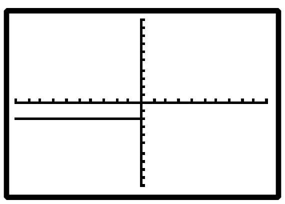
But $\frac{0}{x} = 0$

Therefore, there are no values for x for $\frac{|x-|x||}{x}$ which is a positive integer.

10. *Solution 1*

Using a table of values, we find

x	$f(x)$
–3	$\dfrac{\lvert -3-\lvert -3\rvert\rvert}{-3}=-2$
–2	$\dfrac{\lvert -2-\lvert -2\rvert\rvert}{-2}=-2$
–1	$\dfrac{\lvert -1-\lvert -1\rvert\rvert}{-1}=-2$
0	undefined
1	$\dfrac{\lvert 1-\lvert 1\rvert\rvert}{1}=0$
2	$\dfrac{\lvert 2-\lvert 2\rvert\rvert}{2}=0$



Solution 2

Use a graphing calculator to find the graph. Using the $\boxed{\text{CALC}}$ mode and the $\boxed{\text{VALUE}}$ function, we see $x=0$ gives no answer and is not included in the graph.

Review Exercise

2. **a.** If the x-intercepts are 4, 1, and –2, then $(x-4)$, $(x-1)$, and $(x+2)$ are factors of the cubic function. Therefore, $y=a(x-4)(x-1)(x+2)$, where a is a constant, represents the family of cubic functions.

3. **a.** Let $f(x)=x^5-4x^3+x^2-3$

$$\begin{aligned} f(-2)&=(-2)^5-4(-2)^3+(-2)^2-3\\ &=-32+32+4-3\\ &=1 \end{aligned}$$

Since $f(-2)\neq 0$, $x+2$ is not a factor.

4. Let $f(x)=x^3-6x^2+6x-5$

$$\begin{aligned} f(5)&=5^3-6(5)^2+6(5)-5\\ &=0. \end{aligned}$$

Therefore, $(x-5)$ is a factor of $f(x)$.

By division, x^3-6x^2+6x-5
$=(x-5)(x^2-x+1)$.

5. **a.** Since $(x-1)$ is a factor of $x^3-3x^2+4kx-1$, then $f(1)=0$.

Substituting, $1^3-3(1)^2+4k(1)-1=0$

$$\begin{aligned} 1-3+4k-1&=0\\ 4k&=3\\ k&=\frac{3}{4}. \end{aligned}$$

6. **a.** Let $f(x)=x^3-2x^2+2x-1$

$$\begin{aligned} f(1)&=1^3-2(1)^2+2(1)-1\\ &=1-2+2-1\\ &=0. \end{aligned}$$

Therefore, $(x-1)$ is a factor of $f(x)$.

By dividing, x^3-2x^2+2x-1
$=(x-1)(x^2-x+1)$.

b. Let $f(x)=x^3-6x^2+11x-6$

$$\begin{aligned} f(1)&=1^3-6(1)^2+11(1)-6\\ &=0. \end{aligned}$$

Therefore, $(x-1)$ is a factor of $f(x)$.

By dividing, $x^3-6x^2+11x-6$
$=(x-1)(x^2-5x+6)$
$=(x-1)(x-2)(x-3)$.

7. Since x^2-4x+3
$=(x-3)(x-1)$,

both $f(3)$ and $f(1)$ must be equal to 0 in order to have x^2-4x+3 be a factor of $f(x)=x^5-5x^4+7x^3-2x^2-4x+3$.

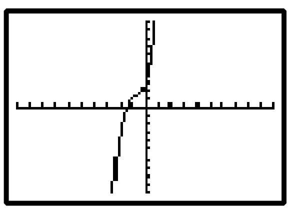
$$\begin{aligned} f(1)&=1^5-5(1)^4+7(1)^3-2(1)^2-4(1)+3\\ &=1-5+7-2-4+3\\ &=0 \end{aligned}$$

Also, $f(3) = 3^5 - 5(3)^4 + 7(3)^3 - 2(3)^2 - 4(3) + 3$
$= 243 - 405 + 189 - 18 - 12 + 3$
$= 0$

Therefore, $(x-1)$ and $(x-3)$ are factors of $f(x)$, and so $(x-1)(x-3)$ or $x^2 - 4x + 3$ is a factor of $f(x)$.

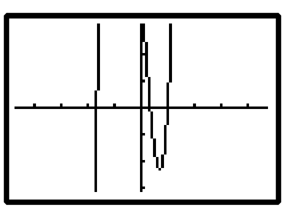
8. **a.** Graphing $y = 2x^3 + 5x^2 + 5x + 3$ yields the graph below. Using the $\boxed{\text{CALC}}$ mode and the $\boxed{\text{VALUE}}$ function, we find when $x = -1.5$, $y = 0$.

Therefore, $f\left(-\frac{3}{2}\right)$ is 0. So, $(2x+3)$ is a factor of $f(x)$.



Therefore, $2x^3 + 5x^2 = 5x + 3$
$= (2x+3)(x^2 + x + 1)$
by division.

b. Graphing $y = 9x^3 + 3x^2 - 17x + 5$ yields the graph below. We can see the x-intercept is between –2 and –1 and perhaps 1. $x = 1$ can be verified by using the $\boxed{\text{VALUE}}$ function in the $\boxed{\text{CALC}}$ mode. Therefore, $x - 1$ is a factor of $f(x)$.



By dividing, $9x^3 + 3x^2 - 17x + 5$
$= (x-1)(9x^2 + 12x - 5)$
$= (x-1)(3x+5)(3x-1)$

The other factors can be tested in the same way as $x = 1$, i.e., let $x = -\frac{5}{3}$ and $x = \frac{1}{3}$.

9. For $f(x) = 5x^4 - 2x^3 + 7x^2 - 4x + 8$,

$f\left(\frac{p}{q}\right) = 0$ if q divides into 5 and p into 8.

a. If $\frac{p}{q} = \frac{5}{4}$, since 5 divides into 5 and 5 into 8,

then it is possible for $f\left(\frac{5}{4}\right)$ to be 0.

b. If $\frac{p}{q} = \frac{4}{5}$, since 4 does not divide into 5, then it is not possible for $f\left(\frac{4}{5}\right) = 0$.

10. **a.** Let $f(x) = 3x^3 - 4x^2 + 4x - 1$

Try $f(1) = 3(1)^3 - 4(1)^2 + 4(1) - 1$
$= 3 - 4 + 4 - 1$
$\neq 0$
$f(-1) = 3(-1)^3 - 4(-1)^2 + 4(-1) - 1$
$= -3 - 4 - 4 - 1$
$\neq 0.$

Therefore, the only binomial factor with integer coefficients must be either $(3x-1)$ or $(3x+1)$. From the graph, we see an x-intercept between 0 and 1, so $(3x - 1)$ is a possible factor.

By division, $3x^3 - 4x^2 + 4x - 1$
$= (3x-1)(x^2 - x + 1).$

b. First, graph $y = 2x^3 + x^2 - 13x - 5$ on your calculator.

We see intercepts $k = \frac{p}{q}$ between –3 and –2, –1 and 0, 2, and 3. Where q divides into 2 and p divides into 5, we try $k = \frac{5}{2}$, $f\left(\frac{5}{2}\right) = 0$.

Therefore, $(2x-5)$ is a factor of $f(x)$.

By division, $2x^3 + x^2 - 13x - 5$
$= (2x-5)(x^2 + kx + 1).$

$= 2x^3 + (-5 + 2k)x^2 + \ldots$

By comparing coefficients, $-5 + 2k = 1$

$+ 2k = 6$

$k = 3$

Therefore, $2x^3 + x^2 - 13x - 5$

$= (2x - 5)(x^2 + 3x + 1)$.

c. Graphing $y = 30x^3 - 31x^2 + 10x - 1$ on your calculator, it can be seen that there is only one value for $k = \frac{p}{q}$ and it lies between 0 and 1. Since q divides into 30 and p into 1, we try $k = \frac{1}{5}$, $f(0.2) = 0$. Therefore, $(5x - 1)$ is a factor of $f(x)$.

By dividing, $30x^3 - 31x^2 + 10x - 1$

$= (5x - 1)(6x^2 + kx + 1)$

$= 30x^3 + (-6 + 5k)x^2 + \ldots$

Comparing coefficients, we have $-6 + 5k = -31$

$5k = -25$

$k = -5$.

Therefore, $30x^3 - 31x^2 + 10x - 1$

$= (5x - 1)(6x^2 - 5x + 1)$

$= (5x - 1)(3x - 1)(2x - 1)$.

11. c. $x^3 + 8 = 0$

$(x + 2)(x^2 - 2x + 4) = 0$

$x + 2 = 0$ or $x^2 - 2x + 4 = 0$

$x = 0$ or $x = -2$ or $x + 5 = 0$

$x = -2$ or $x = \frac{2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$

$= \frac{2 \pm \sqrt{-12}}{2}$

$= \frac{2 \pm 2i\sqrt{3}}{2}$

$= 1 \pm i\sqrt{3}$

Solution set is $\left\{-2,\ 1 \pm i\sqrt{3}\right\}$.

d. $x^3 - x^2 - 9x + 9 = 0$

$x^2(x - 1) - 9(x - 1) = 0$

$(x - 1)(x^2 - 9) = 0$

$(x - 1)(x - 3)(x + 3) = 0$

$x - 1 = 0$ or $x - 3 = 0$ or $x + 3 = 0$

$x = 1$ or $x = 3$ or $x = -3$

e. $x^4 - 12x^2 - 64 = 0$

$(x^2 - 16)(x^2 + 4) = 0$

$(x - 4)(x + 4)(x^2 + 4) = 0$

$x - 4 = 0$ or $x + 4 = 0$ or $x^2 + 4 = 0$

$x = 4$ or $x = -4$ or $x^2 = -4$

$x = \pm\sqrt{-4}$

$= \pm 2i$

f. $x^3 - 4x^2 + 3 = 0$

Let $f(x) = x^3 - 4x^2 + 3$

$f(1) = 1^3 - 4(1)^2 + 3$

$= 0$.

Therefore, $(x - 1)$ is a factor of $f(x)$.

So, $x^3 - 4x^2 + 3 = 0$

$(x - 1)(x^2 - 3x - 3) = 0$, by dividing

$x - 1 = 0$ or $x^2 - 3x - 3 = 0$

$x = 1$ or $x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-3)}}{2(1)}$

$= \frac{3 \pm \sqrt{21}}{2}$.

g. $x^3 - 3x^2 + 3x - 2 = 0$

Let $f(x) = x^3 - 3x^2 + 3x - 2$

$f(2) = 2^3 - 3(2)^2 + 3(2) - 2$

$= 8 - 12 + 6 - 2$

$= 0$.

Therefore, $(x - 2)$ is a factor of $f(x)$.

Note: To select which integer factor to try, first graph $y = f(x)$ and note where the x-intercept lies.

Dividing to find the other factor, we find

$$(x-2)(x^2-x+1)=0$$

$$x-2=0 \quad \text{or} \quad x^2-x+1=0$$

$$x=2 \quad \text{or} \quad x=\frac{1\pm\sqrt{1^2-4(1)(1)}}{2(1)}$$

$$=\frac{1\pm i\sqrt{3}}{2}$$

h. $$x^6-26x^3-27=0$$

$$(x^3-27)(x^3+1)=0$$

$$(x-3)(x^2+3x+9)(x+1)(x^2-x+1)=0$$

$$x-3=0 \quad \text{or} \quad x^2+3x+9=0$$

$$x=3 \quad \text{or} \quad x=\frac{-3\pm\sqrt{9-4(9)}}{2}$$

$$=\frac{-3\pm 3i\sqrt{3}}{2}$$

$$\text{or} \quad x+1=0 \quad \text{or} \quad x^2-x+1=0$$

$$\text{or} \quad x=-1 \quad \text{or} \quad x=\frac{1\pm\sqrt{1-4(1)}}{2}$$

$$=\frac{1\pm i\sqrt{3}}{2}$$

The solution set is $\left\{3,\ -1,\ \frac{-3\pm 3i\sqrt{3}}{2},\ \frac{1\pm i\sqrt{3}}{2}\right\}$.

i. $(x^2+2x)^2-(x^2+2x)-12=0$

Let $\quad a=x^2+2x$

$$a^2-a-12=0$$

$$(a-4)(a+3)=0$$

$$a-4=0 \quad \text{or} \quad a+3=0.$$

But $a=x^2+2x$

Therefore, $x^2+2x-4=0$ or $x^2+2x+3=0$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(-4)}}{2(1)} \qquad x=\frac{-2\pm\sqrt{2^2-4(1)(3)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{20}}{2} \qquad =\frac{-2\pm\sqrt{-8}}{2}$$

$$=-1\pm\sqrt{5} \qquad =\frac{-2\pm 2i\sqrt{2}}{2}$$

$$=-1\pm i\sqrt{2}.$$

Note: Graphing $y=(x^2+2x)^2-(x^2+2x)-12$ confirms the existence of only 2 real roots.

12. c. $x^3-x^2-4x-1=0$

The graph of $y=x^3-x^2-4x-1$ shows 3 real roots between –2 and –1, –1 and 0, and 2 and 3. Using the $\boxed{\text{ZERO}}$ function in $\boxed{\text{CALC}}$ mode, we find $x\doteq-1.377$, $x\doteq-0.274$, $x\doteq 2.651$.

13. If –2 is a root of $x^2+kx-6=0$, where $f(x)=x^2+kx-6$, it means $f(-2)=0$.

Substituting to find k, we get

$$(-2)^2+k(-2)-6=0$$

$$4-2k-6=0$$

$$-2k=2$$

$$k=-1$$

$$\therefore x^2-x-6=0.$$

Also, $(x+2)$ is a factor of $f(x)=x^2-x-6$.

By dividing, the other factor is $(x-3)$.

$$\therefore x-3=0$$

$$x=3$$

So, $k=-1$, and the other root is 3.

14. Let r_1, r_2 be the roots of $2x^2+5x+1=0$. Therefore,

$r_1+r_2=-\frac{5}{2}$ and $r_1r_2=\frac{1}{2}$. The roots of the required equation are $x_1=\frac{1}{r_1}$ and $x_2=\frac{1}{r_2}$. The sum of the new roots is

$$x_1+x_2=\frac{1}{r_1}+\frac{1}{r_2}$$

$$=\frac{r_2+r_1}{r_1r_2}$$

$$=\frac{-\frac{5}{2}}{\frac{1}{2}}=-5.$$

The product of the new roots is

$$x_1+x_2=\frac{1}{r_1r_2}$$

$$=2.$$

Therefore, the new equation is

$$x^2 - (x_1 + x_2)x + x_1x_2 = 0$$

or $$x^2 - (-5)x + 2 = 0$$

$$x^2 + 5x + 2 = 0.$$

15. a. Since

$x^2 -$ (sum of roots) x + (product of roots) = 0,

for $2x^2 - x + 4 = 0$, the sum of the roots

is $\frac{1}{2}$, and the product is $\frac{4}{2}$ or 2.

b. Let x_1, and x_2 be the roots of the quadratic equation

$x_1 + x_2 = \frac{1}{15}$ and $x_1 + x_2 = -\frac{2}{15}$. The equation is

$x^2 - \frac{1}{15}x - \frac{2}{15} = 0$ or $15x^2 - x - 2 = 0$.

c. Let the roots of the quadratic equation be x_1 and x_2.

$$\begin{aligned} x_1 + x_2 &= (3+2i) + (3-2i) \\ &= 6 \\ x_1x_2 &= (3+2i)(3-2i) \\ &= 9 - 4i^2 \\ &= 9 + 4 \\ &= 13 \end{aligned}$$

The required equation is $x^2 - 6x + 13 = 0$.

d.

Solution 1

$3x^2 + 4kx - 4 = 0$ where $f(x) = 3x^2 + 4kx - 4$

If 2 is one root, then $f(2) = 0$.

Substituting, we have

$$\begin{aligned} 3(2)^2 + 4k(2) - 4 &= 0 \\ 12 + 8k - 4 &= 0 \\ 8k &= -8 \\ k &= -1. \end{aligned}$$

Therefore, the equation can be written as $3x^2 - 4x - 4 = 0$. If 2 is one root, then $(x-2)$ is a factor of the function $f(x)$; therefore, $3x^2 - 4x - 4 = 0$ becomes $(x-2)(3x+2) = 0$.

The other root can be found from

$$\begin{aligned} 3x + 2 &= 0 \\ x &= -\frac{2}{3}. \end{aligned}$$

Therefore, the other root is $-\frac{2}{3}$ and $k = -1$.

Solution 2

Let h represent the other root of $3x^2 + 4kx - 4 = 0$.

The sum of the roots is $h + 2 = -\frac{4k}{3}$. (1)

The product of the roots is $2h = -\frac{4}{3}$. (2)

Therefore, $h = -\frac{2}{3}$.

Substituting into (1) to find k,

$$\begin{aligned} -\frac{2}{3} + 2 &= -\frac{4k}{3} \\ -2 + 6 &= -4k \\ 4k &= -4 \\ k &= -1 \end{aligned}$$

e. Let x_1 and x_2 represent the roots of

$x^2 - 5x + 2 = 0.$

$x_1 + x_2 = 5$ and $x_1x_2 = 2.$

The roots of the required equation are $x_1 - 3$ and $x_2 - 3$.

For the new equation, the sum of the roots is

$$\begin{aligned} &(x_1 - 3) + (x_2 - 3) \\ &= x_1 + x_2 - 6 \\ &= 5 - 6 \\ &= -1. \end{aligned}$$

The product of the new roots is

$$\begin{aligned} &(x_1 - 3)(x_2 - 3) \\ &= x_1x_2 - 3(x_1 + x_2) + 9 \\ &= 2 - 3(5) + 9 \\ &= -4. \end{aligned}$$

The required equation is $x^2 + x - 4 = 0.$

f. Let x_1 and x_2 represent the roots of $2x^2 + x - 4 = 0$.

$x_1 + x_2 = -\frac{1}{2}$ and $x_1 x_2 = -\frac{4}{2} = -2$

The roots of the required equation are $\frac{1}{x_1}$ and $\frac{1}{x_2}$.

For the new equation, the sum of the roots is

$$\begin{aligned}\frac{1}{x_1} + \frac{1}{x_2} &= \frac{x_2 + x_1}{x_1 x_2} \\ &= \frac{-\frac{1}{2}}{-2} \\ &= \frac{1}{4}.\end{aligned}$$

and the product is

$$\begin{aligned}\left(\frac{1}{x_1}\right)\left(\frac{1}{x_2}\right) &= \frac{1}{x_1 x_2} \\ &= \frac{1}{-2}.\end{aligned}$$

The required equation is $x^2 - \frac{1}{4}x - \frac{1}{2} = 0$

or $4x^2 - x - 2 = 0$.

16. a. $(x-2)(x+4) < 0$

Solution 1

Graph $y = (x-2)(x+4)$.

Since y is below the x-axis between –4 and 2, therefore, the solution is x such that $-4 < x < 2$.

Solution 2

Consider $(x-2)(x+4) = 0$.

Therefore, $x = 2$ or $x = -4$.

Test:

$x = -5$	$x = 0$	$x = 3$
$\text{L.S.} = (-5-2)(-5+4)$	$\text{L.S.} = (-5-2)(-5+4)$	$\text{L.S.} = (3-2)(3+4)$
$= 7$	$= -6$	$= 7$
But, $\text{L.S.} \not< 0$	$\text{L.S.} < 0$	$\text{L.S.} \not< 0$
$\therefore x \neq -5$	$\therefore x = 0$	$\therefore x \neq 3$

The solution set is $\{x \mid -4 < x < 2\}$.

Solution 3

For the product $(x-2)(x+4)$ to be negative, there are two cases.

Case 1: $x - 2 > 0$ and $x + 4 < 0$
$x > 0$ and $x < -4$

No solution.

Case 2: $x - 2 < 0$ and $x + 4 > 0$
$x < 2$ and $x < -4$

The solution is $-4 < x < 2$.

b. $x^2 + x - 2 \geq 0$

$(x+2)(x-1) \geq 0$

Consider the graph of $y = (x+2)(x-1)$. The values that satisfy the inequality are the values for x for which the y values are on or above the x-axis. The solution is $x \leq -2$ or $x \geq 1$.

c. $x^3 + 3x \leq 0$

$x(x^2 + 3) \leq 0$

Consider the graph of $y = x^3 + 3x$. The solution is those values for x where y is below or on the x-axis, i.e., for $x \leq 0$.

d. $x^3 - 2x^2 - x + 2 > 0$

The graph of $y = x^3 - 2x^2 - x + 2$ is shown with x-intercepts at –1, 1, and 2 as confirmed by using the CALC mode and VALUE function. The solution to the inequality is those values for x where y is above the x-axis, that is $-1 < x < 1$ or $x > 2$.

e. $x^4 \leq 0$

Since x^4 always returns a positive or zero for any value of x, the only solution is $x = 0$. This can be verified graphically by noting that the graph of $y = x^4$ is never below the x-axis.

f. $x^4 + 5x^2 + 2 \geq 0$

Solution 1

From the graph of $y = x^4 + 5x^2 + 2,$ we see that y is always above the x-axis.

Solution 2

Since x^4 and x^2 are always positive, $x^4 + 5x^2 + 2$ is always greater than zero. The solution set is R.

17. a. $|3x - 1| = 1$

Either $3x - 1 = 11$ or $3x - 1 = -11$

$3x = 12 \qquad 3x = -10$

$x = 4 \qquad x = -\dfrac{10}{3}.$

By substituting into the equation, we can verify both answers are correct.

18. The dimensions of the open box are $8 - 2x,\ 6 - 2x,$ and x. The volume is 16 cm^3 or

$$x(8 - 2x)(6 - 2x) = 16$$
$$48x - 28x^2 + 4x^3 = 16$$
$$4x^3 - 28x^2 + 48x - 16 = 0$$
$$x^3 - 7x^2 + 12x - 4 = 0.$$

By graphing $y = x^3 - 7x^2 + 12x - 4$, we find only one real root at $x \doteq 5.11$, but this is an inadmissable root as $x < 3$. Therefore, it is impossible to make a box from this rectangular sheet.

Chapter 2 Test

1. Let $f(x) = x^3 - 5x^2 + 9x - 3$

$$f(-3) = (-3)^3 - 5(-3)^2 + 9(-3) - 3$$
$$= -27 - 45 - 27 - 3$$
$$\neq 0.$$

$\therefore (x + 3)$ is not a factor of $f(x)$.

2. a. $x^3 + 3x^2 - 2x - 2$

Let $f(x) = x^3 + 3x^2 - 2x - 2$

$$f(1) = 1^3 + 3(1)^2 - 2(1) - 2$$
$$= 0.$$

$\therefore (x - 1)$ is a factor of $f(x)$.

By dividing, $x^3 + 3x^2 - 2x - 2 = (x - 1)(x^2 + 4x + 2).$

b. $2x^3 - 7x^2 + 9$

Let $f(x) = 2x^3 - 7x^2 + 9$

$$f(-1) = 2(-1)^3 - 7(-1)^2 + 9$$
$$= -2 - 7 + 9$$
$$= 0.$$

$\therefore (x + 1)$ is a factor of $f(x)$.

By dividing, we find

$$2x^3 - 7x^2 + 9$$
$$= (x + 1)(2x^2 - 9x + 9)$$
$$= (x + 1)(2x - 3)(x - 3).$$

c. $x^4 - 2x^3 + 2x - 1$

Let $f(x) = x^4 - 2x^3 + 2x - 1$

$$f(1) = 1 - 2 + 2 - 1$$
$$= 0.$$

Also, $f(-1) = (1) + (2) - 2 - 1$

$$= 0.$$

$\therefore (x - 1), (x + 1)$ are factors of $x^4 - 2x^3 + 2x - 1.$

By dividing, $x^4 - 2x^3 + 2x - 1$

$$= (x^2 - 1)(x^2 - 2x + 1)$$
$$= (x - 1)(x + 1)(x - 1)(x - 1)$$
$$= (x + 1)(x - 1)^3.$$

3. Graphing $y = 3x^3 + 4x^2 + 2x - 4$ shows one x-intercept between 0 and 1. So, $k = \dfrac{q}{p}$ where p is a divisor of 3 and q is a divisor of 4. Trying $k = \dfrac{q}{3}$ with the $\boxed{\text{VALUE}}$ function in $\boxed{\text{CALC}}$ mode gives $y = 0$.

$\therefore (3x - 2)$ is a factor of $3x^3 + 4x^2 + 2x - 4$.

By dividing, $3x^3 + 4x^2 + 2x - 4$

$$= (3x - 2)(x^2 + kx + 2)$$
$$= 3x^3 + (-2 + 3k)x^2 + \cdots$$

Comparing coefficients, $-2 + 3k = 4$

$$3k = 6$$
$$k = 2.$$

$\therefore\ 3x^3 + 4x^2 + 2x - 4$

$$= (3x - 2)(x^2 + 2x + 2)$$

4. **a.**
$$2x^3 - 54 = 0$$
$$2(x^3 - 27) = 0$$
$$2(x-3)(x^2+3x+9) = 0$$

$x - 3 = 0$ or $x^2 + 3x + 9 = 0$

$$x = 3 \quad \text{or} \quad x = \frac{-3 \pm \sqrt{3^2 - 4(1)(9)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{-27}}{2}$$
$$= \frac{-3 \pm 3i\sqrt{3}}{2}$$

b. $x^3 - 4x^2 + 6x - 3 = 0$

Let $f(x) = x^3 - 4x^2 + 6x - 3$
$$f(1) = 1 - 4 + 6 - 3$$
$$= 0.$$

$\therefore (x-1)$ is a factor of $f(x)$.
$$x^3 - 4x^2 + 6x - 3 = 0$$
$$(x-1)(x^2 - 3x + 3) = 0$$

$x - 1 = 0$ or $x^2 - 3x + 3 = 0$

$$x = 1 \quad \text{or} \quad x = \frac{3 \pm \sqrt{9 - 4(1)(3)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{-3}}{2}$$
$$= \frac{3 \pm i\sqrt{3}}{2}$$

c. $2x^3 - 7x^2 + 3x = 0$
$$x(2x^2 - 7x + 3) = 0$$
$$x(2x-1)(x-3) = 0$$

$x = 0$ or $2x^2 - 1 = 0$ or $x - 3 = 0$

$x = 0$ or $x = \frac{1}{2}$ or $x = 3$

d.
$$x^4 - 5x^2 + 4 = 0$$
$$(x^2 - 4)(x^2 - 1) = 0$$
$$(x-2)(x+2)(x-1)(x+1) = 0$$

$x - 2 = 0$ or $x + 2 = 0$ or $x - 1 = 0$ or $x + 1 = 0$

$x = 2$ or $x = -2$ or $x = 1$ or $x = -1$

5. Let the roots of $x^2 - 2x + 5 = 0$ be x_1 and x_2. The sum is 2 and the product is 5. The roots of the required equation are $x_1 + 3$ and $x_2 + 3$. The sum of the new roots is
$$(x_1 + 3) + (x_2 + 3)$$
$$= (x_1 + x_2) + 6$$
$$= 2 + 6$$
$$= 8.$$

The product of the new roots is
$$(x_1 + 3)(x_2 + 3)$$
$$x_1x_2 + 3(x_1 + x_2) + 9$$
$$= 5 + 3(2) + 9$$
$$= 20.$$

The required quadratic equation is $x^2 - 8x + 20 = 0$.

7. **a.** $(x - 3)(x + 2)^2 < 0$

From the graph of $y = (x-3)(x+2)^2$, the x-intercepts are 3 and –2. y is below the x-axis only for $x < -2$, $-3 < x < 3$, but not for $x = -2$.

b. $x^3 - 4x \geq 0$

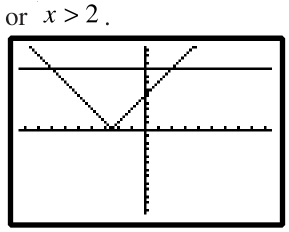
Either $2x - 3 = 7$ or $2x - 3 = -7$
$$2x = 10 \qquad 2x = -4$$
$$x = 5 \qquad x = -2$$

c. $|2x + 5| > 9$

Solution 1

Graph $y_1 = |2x + 5$
$$y_2 = 9$$

The graph of $y_1 > y_2$ for values of x less than –7 and for values of x greater than 9. The solution is $x < -7$ or $x > 2$.



Solution 2

Consider $|2x+5|=9$

Either $2x+5=9$ or $2x+5=-9$

$$\begin{aligned} 2x &= 4 \qquad & 2x &= -14 \\ x &= 2 \qquad & x &= -7. \end{aligned}$$

Take regions to find the solution to $|2x+5|>9$.

$x=-8$	$x=0$	$x=0$
L.S. $=\lvert +2(-8)+5\rvert$	L.S. $=\lvert 2(0)+5\rvert$	L.S. $=\lvert 2(0)+5\rvert$
$=11$	$=5$	$=5$
L.S. >9	L.S. $\not>9$	L.S. $\not>9$
$\therefore x=-8$	$x\neq 0$	$x\neq 0$

Write the answer as $x<-7$ or $x>2$.

8. **a.** The graph shows 3 real zeros at $x=-2$, $x\doteq 1.5$, and $x\doteq 3.5$. The leading coefficient is positive, and the polynomial function is at least cubic, i.e., of degree 3.

b. The graph shows 2 zeros. Since the graph appears to begin in quadrant 2 and to end in quadrant 1, we deduce that the leading coefficient is positive. The shape seems to show a quarter polynomial, i.e., of degree 4.

c. The graph shows 3 zeros. Since the graph appears to begin in quadrant 2 and end in quadrant 4, it will be a cubic function of degree 3 and has a negative leading coefficient.

9. $C=0.0002x^3-0.005x^2+0.5x$

a. Let $x=95$

$$\begin{aligned} C &= 0.0002(95)^3-0.005(95)^2+0.5(95) \\ &= 173.85. \end{aligned}$$

When a diver who weighs 95 kg stands on the board, it will dip 173.9 cm.

b. If the diving board dips 40 cm, $C=40$.
Substituting, $40=0.0002x^3-0.005x^2+0.5x$
or $0.0002x^3-0.005x^2+0.5x-40=0$
Graphing $y=0.0002x^3-0.005x^2+0.5x-40$ and using the $\boxed{\text{ZERO}}$ function in $\boxed{\text{CALC}}$ mode, we find $x\doteq 51.6$. So, the diver has a mass of about 52 kg.